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THE VALUATION
OF
WIDOWS' FUNDS.

— — —
DAVID R.W. HUIE

ECMC (Reserve Stoll.)

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THE VALUATION
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WIDOWS' FUNDS.

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REMARKS
ON THE
VALUATION OF WIDOWS' FUNDS

WITH
TABLES
TO ASSIST THE ACTUARY IN SUCH INVESTIGATIONS,
BASED ON THE EXPERIENCE OF THE WIDOWS' FUNDS OF THE
MINISTERS AND SCHOOLMASTERS IN SCOTLAND.

BY
DAVID R. W. HUIE,
MEMBER OF THE FACULTY OF ACTUARIES IN SCOTLAND.

EDINBURGH:
R. GRANT & SON, 54 PRINCES STREET.
LONDON: CHARLES AND EDWIN LAYTON.

1868.

PRINTED BY NEILL AND COMPANY, EDINBURGH.

1210177-190

PREFACE.

IN the early part of the session of 1864, I read a paper before the Actuarial Society of Edinburgh, on the Valuation of Widows' Schemes, and on that occasion I explained the mode of preparation of a few Tables, arranged from the statistics of the Widows' Funds of the Ministers of the Church of Scotland, and of the Burgh and Parochial Schoolmasters in Scotland, exhibiting the law of marriage among the members of these Societies. It occurred to some of the gentlemen present on that occasion, that the publication of these Tables might be useful to Actuaries engaged in the examination of the affairs of such bodies, and I was requested to extend my paper and give it publicity. This, then, is my excuse for offering such an humble work to the profession. I publish it nearly in the same state in which it was originally written, as other engagements have prevented my devoting much additional time to its completion. Having been prepared for a Society, many of whose members had but recently turned their

attention to actuarial subjects, there was necessarily much of elementary matter introduced, but I trust that the Tables may be found of service to some Actuaries. I do not know of any already in existence which are exactly similar ; at the same time, I do not claim great originality for the idea, as some of them were suggested to me by the perusal of a volume of manuscript tables left by the late eminent Actuary, Griffith Davies, to which I shall afterwards refer in describing my own Tables.

I take this opportunity of expressing the obligations under which I lie to Mr Thomson, the Manager of the Standard Life Assurance Company, for placing at my disposal much valuable statistical matter which he had collected during his actuarial career. Indeed, almost all the statistics on which the following Tables are founded were supplied me by Mr Thomson.

D. R. W. H.

REGISTER PLACE, EDINBURGH,
6th March 1868.

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REMARKS

ON THE

VALUATION OF WIDOWS' FUNDS.

BEFORE the modern system of Life Assurance was introduced, there was no means by which a husband and father could provide an independence for his family in the event of his early death, unless he belonged to some guild or corporation which possessed sufficient funds to offer relief to the destitute families of its members.

The first effort in the direction of life assurance was the establishment of Widows' Funds.

When these Societies were originally founded the true principles on which they should be established were not understood, and, consequently, great difficulty was experienced in adjusting the rates payable by the members to secure the benefits of the fund. Having no Mortality Tables to assist them, the founders of these Societies endeavoured by an elaborate calculation to ascertain how many widows were likely to come upon the Fund every year, and at what time the number of widows would reach the maximum, after which period the numbers might be expected to remain stationary, or nearly so. These numbers being ascertained, it next became their object to provide an annual fund, sufficient to meet the yearly-increasing amount of annuities, and at the same time to accumulate a capital, the interest of which would, along with the annual rates, be sufficient to pro-

vide annuities for all the widows when their numbers should have reached the maximum.

In calculating the annual payments from the members of these early Societies, it was the number of probable contributors, not their respective ages, that was the chief object of consideration ; and hence we find that an equal annual payment was made by every member, whatever his age might be, or whatever might be the disproportion between his age and that of his wife. Bachelors also paid the same rates as married men.

It is evident that a Society constituted on the maximum principle, where the liability entailed on the fund by *each individual* was not estimated, but only the probable liabilities following upon a constant succession of individuals, could only be successfully conducted upon one condition, namely, that the number of members remained stationary, and that, consequently, the number of widows would reach a point at which they also would neither increase nor decrease.

The interest of the capital alone being divisible, it necessarily followed that, if the number of members, and ultimately of widows, greatly increased, the income must eventually prove inadequate to meet the annuities ; while, on the other hand, should the number of members be considerably diminished, the number of widows would in time fall greatly below the estimated maximum, and, consequently, there would be a surplus income, which would go to swell the capital to a wholly unnecessary extent.

The Widows' Fund of the ministers of the Church of Scotland was founded on the maximum principle. It has been eminently successful, because in its history the one necessary condition above mentioned has been fulfilled. The number of contributors remained nearly unaltered for the first century after its establishment ; for during that period the number of parishes experienced little change, and each minister was obliged by Act of Parliament to contribute to the Fund, and to maintain his annual payment as long as he lived.

As the following Tables are in a great measure based on the

experience of the Ministers' Widows' Fund of the Church of Scotland, it may not be out of place to give a short account of that Society.

The proposal for a general Widows' Fund for the whole Church was first brought forward in 1742. The want of such a scheme had long been felt, and various local societies had been set on foot to make provision for widows and orphan families left destitute by the untimely death of the head of the house—but these had all failed from various causes.

The scheme, as at first originated, left it optional with the various clergymen to join or not as they thought proper, and it is worthy of notice that 85 per cent. of the then existing holders of benefices immediately signified their intention of becoming contributors. The Act of Parliament constituting the Society rendered a contribution to its funds imperative on all who should afterwards obtain appointments in the Church.

Owing to various circumstances, and to the fact that there had been some errors in the original returns of the numbers of ministers and widows upon which the calculations had been based, it was found in a few years that there was danger of the funds proving insufficient to meet the necessary charges, and, consequently, one or two modifications were suggested, and authorised by Act of Parliament in 1748.

After this the funds of the Society proved amply sufficient to meet all claims, and no alteration of any importance took place until 1813, when it was proposed that, on account of the increased expense of living, it was expedient to increase the amount of the annuities; and as it was necessary for this purpose to levy an additional annual rate from the contributors, another Act of Parliament was obtained, by authority of which not only was provision made for an immediate increase to the annuities, but the foundation of an additional fund was laid, which has enabled the Trustees to augment the annuities to a further extent from time to time.

Other societies, founded on the pattern of the Ministers' Widows' Fund, have not been so happily conducted. Not having

been able to maintain a constant number on their lists, they have fallen into one or other of the difficulties already referred to. Some have been obliged to wind up from insufficiency of funds, while others have found their capital increasing far beyond their wants, and have ultimately raised their annuities to a point much above what was originally contemplated.

This may not, at first sight, appear an evil, but it is so, nevertheless. The increase to the annuities has seldom taken place until those whose contributions actually raised the capital have long ceased to derive any benefit from it; and, consequently, the widows of the later entrants were enabled to draw annuities out of all proportion to the contributions paid, the excess having been obtained at the expense of their predecessors.

The maximum system, then, was a false one. Every Society founded on this principle was certain to meet with great variations in its prosperity, so long as the number of its members could not be maintained at the same amount. And the injustice of making all the contributions equal, notwithstanding the difference of age and condition, is so apparent, that no society would now be established on any other principle than that adopted by assurance offices, namely, that each person should pay an exact equivalent for the liability his entrance entails upon the fund. His annual contribution should vary according to his age, and according to the probability of his marriage, if he is a bachelor; and at his marriage, the age of his wife should form a material element in the calculation.

Tables are now at the Actuary's disposal which facilitate the calculation of almost every possible contingency depending on the duration of human life; but tables showing the probability of marriage, and the value of the consequent liabilities, are not so readily obtained.

I suppose that every Actuary who has been engaged in investigating the affairs of Widows' Funds must have experienced considerable difficulty in forming an opinion as to the liabilities resulting from first and second marriages, and also in estimating the value of payments to children, whether in the form of

annuity or in a single payment. There are no Tables that I am aware of which give information at all conclusive upon these subjects. The Marriage Table given by Mr Cleghorn at the end of his work on Widows' Funds, has formed the basis of many subsequent calculations; but, as I shall afterwards explain, this Table was formed upon imperfect data.

During a late valuation of the Widows' Funds of the Ministers of the Church of Scotland, and of the Burgh and Parochial Schoolmasters of Scotland, in which I was engaged in assisting Mr William Thomas Thomson, we became possessed of a quantity of statistics so extensive, that it was deemed inexpedient to lose the opportunity of forming Tables of Marriage, &c., for future guidance. The time required to prepare these Tables was, however, so great, that they were not attempted before the publication of Mr Thomson's Report on the Ministers' Widows' Fund in 1863; but since the preparation of that Report, Mr Thomson has placed those statistics at my disposal, along with some other very valuable information in his possession on the same subject; and from these sources I have prepared the few Tables, which I shall afterwards endeavour to explain, and which I now publish, in the hope that they may be found useful by other Actuaries engaged in similar investigations.

In using them, however, the Actuary will keep in mind that there is always some danger in applying the statistics furnished by one profession to estimate the liabilities resulting from the marriages in another.

The safest mode of valuing a Widows' Fund is to base the calculations on the facts of its own history. But few such societies are of sufficient extent to furnish a sufficiency of facts; and in many there has not been proper care taken to record the necessary information. In such cases, I believe the following Tables will be a safe guide; for I do not think that the marriage rate there shown will be exceeded in many Societies.

The average age at marriage among the individuals composing any trade or profession will be found to depend very much upon the age at which the members generally attain what appears

to them a fair income, sufficient to maintain a family upon, and which is not likely to be much increased. The less remunerative the employment the sooner will this point be reached, and hence we find that among the labouring classes marriages take place at an earlier age than among the higher classes of society, at least among that portion of the higher classes whose income is dependent upon their own exertions. And when sufficient statistics are obtained it will be found, I think, that marriages likewise take place at an earlier age among the members of those professions whose income is a fixed one—such as clergymen, schoolmasters, &c.—than among individuals of such professions as the legal and medical, where the income is fluctuating, and where what is considered a competent income to settle upon is not obtained until after many years of probation and tedious uncertainty. The ages of the wives at marriage will also vary in different cases from similar causes, for the longer the marriages are deferred the greater will be the difference between the age of husband and wife. These points are of some importance, and the Actuary should direct his particular attention to them before proceeding with his valuation of assets and liabilities. The operation of this rule will, of course, be more particularly observed in such Widows' Funds as are composed of the members of one profession, and when it is compulsory upon the members of that profession to join the Fund. When admission to the Fund is optional, I should imagine that few would join who have not an intention of marrying, and, therefore, in such societies the marriage rate will be higher, and the marriages will also probably take place soon after admission.

Before commencing the investigation, the Actuary must settle, not without mature consideration, two very important points, namely, the rate of interest to be assumed upon the society's investments, and the table of mortality to be used as the basis of his calculations.

It is now an almost invariable rule to value the liabilities of an assurance office on the basis of the Carlisle Table, with 3 per cent. interest, and few Actuaries will be found to recommend a

higher rate in such calculations. There is no doubt that the past experience of every office shows a more favourable return, and there seems little probability that even the highest class of securities, in which alone assurance offices are presumed to deal, will continue to yield a higher rate. It is true that the amount of money seeking investment is year by year increasing, and must go on increasing in proportion to the prosperity of the country. But the progress of civilisation creates new demands for money, and the immensely increased facilities of communication between distant parts of the globe are yearly causing increased demands upon the capital of the country, so that there does not appear to be much cause to apprehend an unfavourable change in the average value of money as measured in itself, whatever the ultimate effects of the continued influx of gold may be on the value of money as measured in other articles. *not*

But while the Actuary may see no reason to apprehend much change in the rate of interest from its present state, he must bear in mind that the period embraced in his calculations of future risks is very great. In commencing an investigation he has to consider what will be the rate of interest realised until the extinction of the last life now connected with the Fund, or to be connected with it through the subsequent marriage of any of its present members. A moment's consideration will show that this embraces a very long period of years, during which there may be many important changes in the political and commercial world which it is at present impossible to foresee, and which may powerfully contribute to affect the rate of interest as well as the security of the capital invested. Hence the prudence and necessity of adopting a low rate of interest in his calculations, particularly if the society is of considerable extent. If the capital is small, it will be the easier to find profitable investment for it; but, on the other hand, a small society is more exposed to fluctuation in its mortality experience. Even in a very limited Society, therefore, it is inexpedient to run any risk, or to be over-sanguine in calculating on a favourable return on the investments.

But the Actuary will be guided very much in his conclusions on this point by the class of securities pointed out by the rules of the Society, and by the rate of interest hitherto realised on its investments. It is possible that he may not consider it necessary to adopt the minimum rate of 3 per cent. Yet I question if it will often be prudent to take a higher rate than $3\frac{1}{2}$ per cent., clear of Income tax.

The rate of interest is, however, so important an element in the calculations, that it must be most carefully considered, as one-half per cent. will make a very material difference in the results of the investigation.

The Table of Mortality to be followed is another matter requiring careful attention.

In valuing the affairs of Widows' Funds, where the risks depend upon a nearly equal number of both sexes, the difference in value of male and female life becomes an important consideration; and unless allowance is made for the superior longevity of the female portion of the community, the risk of error is considerable.

As many Widows' Funds are composed of individuals belonging to a particular trade or profession, it is quite possible that an instance may occur where the male and female lives are nearly equal in value; but this is not likely to be frequently the case. Probably, in every instance the proportion will vary to some extent, which can only be ascertained by a careful study of the statistics furnished by the past history of the Society.

In the Carlisle Table there is no distinction between the sexes, and this is a serious drawback to its employment in the valuation of Widows' Funds. It is, however, the Table with which Actuaries are most familiar, and notwithstanding its faults of imperfect gradation in the centre, and excessive attenuation at the close,* it is likely to continue to be the most extensively

* It does not seem at all probable that the latter portion of the Carlisle Table will be borne out by any experience. The nearest approach to it that has come under my own observation was in the case of an investment in Government Life Annuities, made by some of the Scotch Insurance Companies and Banks about the year 1832. At that time the National Debt Commissioners

employed, if for no better reason, at least on account of the immense amount of tabular matter based upon it, embracing calculations of nearly every description. All experience, also, goes to show that it is upon the whole a very fair exponent of the general mortality of the country. There are other Tables, such as the Government Annuity experience, the English Life Tables, the Swedish Life Table, &c., where the mortality among males and females is separately given, but the objection stated above to the use of the Carlisle Table applies in a measure to them also. For while the Carlisle Table values male and female lives alike, the difference in value shown by either of the other Tables mentioned may be very far from agreeing with what is indicated by the experience of the Society, and consequently, the risk of error may be as great, though perhaps in a contrary direction. The only safe course for the Actuary is to study carefully the statistics of the Society (if they can be made available), and to endeavour to ascertain to what published table of mortality they make the

made no restrictions as to the age of the selected lives on which they granted annuities, and the nominees I refer to were all chosen between the ages of 77 and 80. The following is the number surviving each year from 80 to the end of life. In a parallel column I have placed the figures which the Carlisle Table would afford at corresponding ages.

AGE.	Annuity Nominees.	Carlisle Table.	AGE.	Annuity Nominees.	Carlisle Table.
80	70	70	92	7	5.51
81	61	61.48	93	4	3.97
82	58	53.25	94	2	2.94
83	52	45.76	95	2	2.20
84	46	38.86	96	2	1.69
85	39	32.69	97	2	1.32
86	32	26.96	98	1	1.03
87	28	21.74	99	1	.81
88	24	17.04	100	1	.66
89	16	13.29	101	1	.51
90	12	10.43	102	1	.36
91	8	7.71	103	0	.21

The figures here given are very remarkable, and it is almost unnecessary for me to add that the transaction proved a very profitable one to the parties who engaged in it. Since the period mentioned, the operation of the Income tax, and the refusal of the National Debt Commissioners to receive selected lives above a certain age, have combined to reduce the profits of such investments. But with judicious choice of nominees it would appear that $4\frac{1}{2}$ per cent. may still be safely calculated on.

nearest approach. If the death rate shown by the statistics of both sexes does not so closely coincide with that of any known Table, where there is a distinction of sex, as to clearly point to its adoption as a basis, or if the records of the Society are not sufficiently extensive to enable him to form a decided conclusion from them, the Actuary will probably proceed upon the basis of the Carlisle Table, adding, however, such a percentage to the amount of liabilities as may appear called for by the increased value of the female lives. A course similar to what is now recommended was adopted in a recent investigation into the affairs of the Ministers' Widows' Fund of the Church of Scotland. The records of that large Society did not, previous to 1863, afford sufficient data from whence to deduce the mortality experienced by it, and as there was a question as to whether the mortality among parish ministers might not be considerably under that observed among the general population of the country, the profession of a clergyman being usually considered one very conducive to health and longevity, it was deemed advisable to collect statistics, and to form a table of mortality in which the sexes might be distinguished, and which might itself form the basis of the intended valuation. The difficulties encountered in this process need not be mentioned here, they will be fully appreciated by any who have ever made a similar attempt, and by all who know the loose manner in which the entries of deaths were made in our parish records before the passing of the Registration Act. It is enough to say that, after great labour and patience, the ages of nearly all the clergymen and widows connected with the Fund who died in the eighteen years preceding 1863 were ascertained. Full particulars as to age, &c., of all the surviving members had previously been obtained by means of schedules issued to them. The following Table of Mortality was then constructed. This Table appeared in Mr Thomson's Report on the Ministers' Widows' Fund, published in 1863, from which I have copied it with Mr Thomson's permission.

Comparative Table showing the Yearly Mortality at each Age, according to the Carlisle Mortality Table, and according to the results in the Ministers' Widows' Fund, also the Expectation of Life deduced from these Tables.

AGE.	CARLISLE TABLE.		MINISTERS' WIDOWS' FUND.				EXPECTATION OF LIFE.		
			Males.		Females.		CARLISLE.	MINISTERS' WIDOWS' FUND.	
	Living.	Dying.	Living.	Dying.	Living.	Dying.		Males.	Females.
44	4798	71	5234	25	4798	92	25.09	25.35	26.38
45	4727	70	5209	16	4706	90	24.46	24.47	25.89
46	4657	69	5193	71	4616	87	23.82	23.55	25.38
47	4588	67	5122	39	4549	59	23.17	22.86	24.75
48	4521	63	5083	62	4490	...	22.50	22.04	24.07
49	4458	61	5021	46	4490	46	21.81	21.31	23.07
50	4397	59	4975	85	4444	44	21.11	20.50	22.30
51	4338	62	4890	69	4400	90	20.39	19.85	21.52
52	4276	65	4821	77	4310	83	19.68	19.12	20.96
53	4211	68	4744	79	4227	40	18.97	18.42	20.36
54	4143	70	4665	157	4187	88	18.28	17.73	19.55
55	4073	73	4508	128	4149	...	17.58	17.33	18.73
56	4000	76	4380	83	4149	64	16.89	16.82	17.73
57	3924	82	4297	85	4085	63	16.21	16.14	17.00
58	3842	93	4212	78	4022	93	15.55	15.45	16.25
59	3749	106	4134	116	3929	61	14.92	14.73	15.63
60	3643	122	4018	135	3868	59	14.34	14.14	14.87
61	3521	126	3883	86	3809	61	13.82	13.62	14.09
62	3395	127	3797	99	3748	119	13.31	12.91	13.31
63	3268	125	3698	177	3629	114	12.81	12.25	12.73
64	3143	125	3521	98	3515	112	12.30	11.84	12.13
65	3018	124	3423	132	3403	191	11.79	11.16	11.52
66	2894	123	3291	177	3212	132	11.27	10.59	11.17
67	2771	123	3114	156	3080	101	10.75	10.16	10.63
68	2648	123	2958	146	2979	185	10.23	9.67	9.97
69	2525	124	2812	194	2794	109	9.70	9.15	9.61
70	2401	124	2618	159	2685	316	9.18	8.79	9.13
71	2277	134	2459	147	2369	86	8.65	8.33	8.98
72	2143	146	2312	234	2283	124	8.16	7.83	8.46
73	1997	156	2078	151	2159	100	7.72	7.65	7.92
74	1841	166	1927	179	2059	168	7.33	7.21	7.29
75	1675	160	1748	181	1891	245	7.01	6.90	6.90
76	1515	156	1567	120	1646	100	6.69	6.63	6.85
77	1359	146	1447	106	1546	80	6.40	6.15	6.26
78	1213	132	1341	113	1466	142	6.12	5.59	5.58
79	1081	128	1228	120	1324	194	5.80	5.06	5.12
80	953	116	1108	204	1130	221	5.51	4.55	4.91
81	837	112	904	190	909	149	5.21	4.47	4.98
82	725	102	714	61	762	94	4.93	4.53	4.84
83	623	94	653	113	668	62	4.65	3.90	4.43
84	529	84	540	100	606	102	4.39	3.61	3.84
85	445	78	440	95	504	98	4.12	3.32	3.52
86	367	71	345	79	406	84	3.90	3.10	3.25
87	296	64	266	61	322	73	3.71	2.88	2.96
88	232	51	205	55	249	60	3.59	2.58	2.69
89	181	39	150	40	189	55	3.47	2.35	2.38
90	142	37	110	33	134	42	3.28	2.02	2.15
91	105	30	77	28	92	33	3.26	1.67	1.90
92	75	21	49	22	59	25	3.37	1.34	1.69
93	54	14	27	17	34	14	3.48	1.02	1.56
94	40	10	10	7	20	11	3.53	.90	1.30
95	30	7	3	2	9	5	3.53	.83	1.28
96	23	5	1	1	4	2	3.46	.50	1.25
97	18	4	2	1	3.28	...	1.00
98	14	3	1	1	3.0750

I quote the Table here just as it was constructed, and without attempting to graduate it so as to fill up the blanks which here and there occur, and cause the line of decrement to run more smoothly. A moment's glance will show how nearly the male expectation coincides with the Carlisle Table, which was therefore adopted as the base of the calculations. The expectation of female life shows a somewhat higher value than the Carlisle Table until the concluding years are reached. A percentage was therefore added to the value of future annuities, corresponding to the observed difference in value in the two classes of lives. The above Table is interesting, although the number of deaths actually observed (about one thousand) is perhaps not sufficiently great to warrant the assumption that the mortality among clergymen and their wives is here faithfully represented, still I have no doubt it is a close approximation to the truth, and is additional evidence of the correctness of the Carlisle Table. I have not quoted the first part of the Table for this reason. The observations having necessarily embraced new entrants on the Fund as well as old members, I am under the impression that this would to some extent reduce the mortality of the early years, as we must, I think, conclude that those clergymen who are appointed to livings are in a sense selected lives. It is but reasonable to suppose that no one would be chosen for parish duty who was not in the enjoyment of comparatively good health. Hence we must expect to find the mortality during those years, when there are a number of new entrants, considerably reduced, just as the mortality experience of an assurance office, where the new lives are not excluded, will appear much more favourable than a more correct investigation would show. The average age at appointment of ministers of the Church of Scotland having been observed to be about 32, I conclude that at age 45 the advantage of selection must be nearly exhausted, and the new members after that age are too few to materially affect the results.

Few Widows' Funds will afford data sufficient for the construction of a mortality table such as the above; but they will generally furnish materials which will enable the experienced

Actuary to come to a satisfactory conclusion on the subject, and to indicate to him the table of mortality he should employ.

The following Tables are founded on the Carlisle experience. I believe that on this foundation they will be more useful and more generally applicable than on any other.

The Actuary having decided on the rate of interest and table of mortality, will next proceed to arrange the various particulars of each case connected with the Society, in such a way as will enable him to extract the utmost information with the least possible trouble. A little attention to the arrangement will often save a considerable amount of labour; and I might here suggest that the members should be separated into "married," "single," and "widowers," and again subdivided according to the amount of their annual contribution.

The particulars of each case should also be arranged in parallel columns, which will save a very considerable amount of trouble, as by the summation of these columns the totals and averages of each head may readily be ascertained. The examples which are given in the Appendix will convey this idea more thoroughly, and will serve to show what particulars are generally required to be observed in these investigations.

The assets of a Widows' Fund may be divided into two classes—those which are already realised, and those which are contingent and prospective. In regard to the first class, I need make no remark. It is not usually required of the Actuary to give his opinion on these. The proper value to be put upon them will be best known to the managers or trustees of the Society. But with regard to the second class, the Actuary alone is the proper judge. The contingent assets in most Funds will consist of—

1. The value of future contributions from members;
- 2. The value of marriage fines;
3. The value of equalising taxes.

The future contributions from members will be very easily valued when the regulations of the Widows' Fund require the same contribution from all, whether married or unmarried, and

without any variation in respect of age. In such a case, all that is required in order to obtain the amount of future annual payments, is to multiply the total summation of the columns in each class containing the value of £1 per annum on each male life by the annual subscription payable by the members of that class. One multiplication will therefore take the place of a great many. But when the rules of the Society require only a small payment from the members as long as they remain unmarried, to be increased at marriage to the annual sum which may then appear equivalent to the liability incurred, the valuation of future contributions becomes a more complicated matter ; and various disturbing elements are introduced into the calculations, such as the possibility that the unmarried member may never increase his subscription by marriage, the uncertainty of the age at which he may marry, and the difficulty of fixing the age of the wife he may take ; then the increased contribution of married members may be calculated to be payable only during the currency of subsisting marriages, to be reduced in the event of the wife's death, and to be rearranged at any subsequent marriage. In fact, a variety of contingencies may require to be estimated, which will depend on the rules of the particular Society. The tables of "Average Age at Marriage," "Probability of Marriage," "Average Age of Wife," will, however, greatly facilitate these calculations.

In any case, when the date of the valuation does not fall immediately after the payment of the member's subscription, it must be borne in mind that an addition to the amount of future payments falls to be made on account of the portion of the year which has elapsed since the *last* payment fell due. This, of course, is on the supposition that the ordinary annuity tables are used, when the first payment is assumed to be due six or twelve months hence. If immediate annuities are employed, the proportion until the date of *next* payment falls to be *deducted*. This simple rule is sometimes neglected in the valuations of assurance societies, and the error will vary according to circumstances, and may sometimes be very considerable.

There is a point of some importance to be considered in valuing future contributions, namely, that in *optional* Societies a good many resignations may be expected to take place; and as it is not the practice of such Societies to allow any surrender value on retirement, the profits to the Fund may be considerable. But the past experience of the Society can alone indicate the probable amount of the profit, and in many cases it may be considered better to leave it unvalued, and let it stand as an offset to other contingencies.

Marriage fines have been introduced into those Societies where all members, whether married or unmarried, pay the same contribution (varied only according to the benefit secured), and they are intended to correct any injustice in such a scheme. Tables will be found at the end of this volume to facilitate their valuation. These Tables, of the value of £1, payable at marriage, will also be useful in estimating the value of equalising taxes, when such are provided by the regulations of the Society. This tax is another attempt to correct the injustice of equal payments by all members. It is evident that a much heavier liability is incurred by the Fund in consequence of the members marrying wives who are greatly their juniors, than if husband and wife were of nearly equal ages. The marriage tax is therefore proportioned to the difference between the age of husband and wife.

Such attempts to equalise the payments by the different members are, however, very rude and ineffectual, and were adopted at a time when actuarial science was not understood; and almost every Society, whose contributions are regulated on this principle, has been obliged at various times to modify its arrangements, and to alter either the amount of subscriptions levied from the members, or the amount of annuity payable to the widows.

Many Widows' Funds, particularly those founded on the maximum principle, are bound, by their constitution, to divide only the annual income from whatever source, and by no means to encroach upon the capital. In those cases where the member's payments are a fair equivalent for the benefits received, and the capital is purely the result of the accumulation of these

payments, such a stipulation will make no difference in the valuation. The capital may be taken at its full amount, and considered entirely applicable to the liquidation of the liabilities connected with the members at the time of the investigation, for in such cases the liabilities connected with future members will be provided for by their contributions.

But when the capital has been increased by any adventitious circumstance above the natural result of the accumulation of contributions, and the member's payments are manifestly insufficient in themselves to provide the annuity which has been divided, the whole of the existing capital should unquestionably not be taken into the account of assets. In strict justice, the accidental capital must be considered as a fund for the benefit of all future members, in the income from which each member has only a life interest; and if it were all taken into the amount of assets at any investigation, and the annuities modified accordingly, this would simply be appropriating it entirely for the benefit of the then existing members and their families. When such a case is laid before him, the Actuary should make this plain to the trustees of the Fund; for although the annuities might be temporarily increased, the result would most assuredly be either an encroachment on the capital, or a large reduction of the annuities at no very distant date.

There are some Widows' Funds which, besides the annual payments from members and the interest of vested capital, have an additional income from other sources. The valuation of such an asset is also a matter of some difficulty. It is manifest that it cannot be capitalised and all thrown in for the increase of the assets at each investigation. As in the former case, the members have only a life interest in it, and that interest should rather be valued as a decreasing annuity, which will expire on the extinction of the last life now connected with the Fund.

The assets having been valued, the next consideration is the amount of liabilities. These, in the generality of cases, will consist of—

1. Annuities to widows on the lists.
2. Annuities to widows who may be left from subsisting marriages.
3. Annuities to the widows who may be left through the subsequent marriage of members now living in wedlock.
4. Annuities to widows who may be left through the remarriage of widowers.
5. Annuities to the widows of members presently unmarried.
6. Annuities to the widows who may be left through their subsequent marriages.

Besides these now enumerated, the rules of the Society may introduce other contingencies. There are many Funds now existing which make provision for orphan children, either as an annuity for a term of years, or as a single payment. But as the arrangements regarding children differ in almost every Fund with which I am acquainted, and as the information I have been able to obtain in regard to the birth of children has not been sufficiently conclusive, I have not attempted to construct many Tables to assist in valuing such contingencies.

The value of future payments to widows, who are on the lists at the time of investigation, will be a very simple calculation, being the annual sum receivable by each widow multiplied into the value of an annuity at the age she has attained. If the widows are separated into classes, as recommended in the case of the members, only one multiplication will be necessary for each class. Care must be taken in any case to make the usual additions for half-yearly payments, if the annuities are payable twice a-year; and a further addition must be made in all cases for the proportion of the year which has elapsed since the last payment was made. Should the payments cease on the annuitant marrying again, some allowance will require to be made in the calculation. I am not aware that there are any statistics existing which will give satisfactory evidence of the value of this contingency.

The value of annuities to widows who may be left from mar-

riages subsisting at the date of examination, is the value of a survivorship annuity to the wives then living after the death of their husbands. The rule for attaining this value is, to deduct from the value of an annuity on the wife's life the value of an annuity payable while they are both living. The arrangement of the members into classes will be of material assistance here in shortening labour. It will not be necessary to make a separate calculation for each case; but the summation of the column containing the value of £1 on the joint life can be deducted from the summation of the values of the annuities on the female lives, and the difference, multiplied by the annuity secured to the members of that class, will give the required value.

Annuities payable to the widows who may be left through the subsequent marriage of members living in wedlock at the time of investigation, may be valued with the assistance of Tables IX. to XVI. attached to this work. The numbers opposite each year of age represent the value of an annuity of £1, to be paid in the event of the member becoming a widower, and afterwards marrying a second wife, who shall survive him. The construction of these Tables will be afterwards explained.

It will be observed that these Tables are only calculated to a difference of seven years between the ages of husband and wife. This appeared to me to be quite sufficient for all practical purposes; for, the liability under this head being comparatively small, I have found that sufficient accuracy is obtained by taking the average difference between the ages of the married members and their wives, and in each case employing the values given in the Table corresponding to this difference. Thus, if the average difference of age were found to be six years, it would answer perfectly well to employ only Table XV., and in each case to take the values corresponding to the husband's age in this Table. The labour saved in this way is very considerable.

Tables are also supplied for the valuation of annuities payable to widows who may be left through the remarriage of widowers. The probability of a widower marrying is much greater than of a bachelor of equal age. Indeed, the number of young widowers

who do not marry a second time, as shown by the experience of the two Funds on which the Tables are based, is little more than ten per cent. The ages of the wives selected by them are also somewhat different from the ages of first wives, but this point will be again referred to in the description of the Tables.

The liabilities resulting from the first and second marriage of bachelors can be ascertained by using Tables VI. VII. and XVII. I have constructed two Tables for valuing annuities to widows of first marriages of men now unmarried—one founded on the experience of the Ministers' Widows' Fund, and the other on the experience of the Schoolmasters' Widows' Fund. The number who marry once out of 2000 persons is very much the same in both Tables, but the proportion who marry at each age is different. In the Schoolmasters' experience, one-half of the first marriages take place before the age of thirty-one, while by the Ministers' experience this division is not reached until thirty-four. The ages of their wives were found to coincide very closely in both Societies. The average age at marriage, among the members of the Fund which he is investigating, will indicate to the Actuary which of these two Tables to employ.

The various heads now noticed embrace nearly all the liabilities which can be brought upon the Fund through the survival of widows. Third and fourth marriages are of so very rare occurrence that the value of annuities dependent on such contingencies would be very small. It was therefore not considered expedient to attempt to construct Tables for their valuation; indeed, the basis of such Tables would have necessarily been almost entirely imaginary, for out of 600 deceased members of the Ministers' Widows' Fund, whose marriages were traced, only two were found to have married three times. One married at the age of 52; the age of the other was not discovered. Only *one* out of 1305 members of the same Society living at November 1861 was thrice married; this marriage took place at the age of 55. Of these 1305 members, 369 had passed the age of 60. In the Schoolmasters' Widows' Fund, only two out of 1256 members living at August 1859 were thrice married. It is evident from

these figures, that the number of third marriages is very small; and it is equally evident that no law can be deduced from them. The late Mr Cleghorn estimated that one person out of 100 would marry three times, but the percentage is no doubt far less than this.

It has been already remarked that some Widows' Funds make provision for orphan children, either in the form of annuity or as a single payment. When the provision takes the form of an annuity, it is usually continued until the youngest child attains a certain age—as 18 or 21—but payable only in the event of both father and mother dying while the children are under these ages. In valuing these annuities, they may be divided into five classes, payable to

Children whose parents are both dead.

Children of widows.

Children of widowers.

Children whose parents are both alive.

Children of members who are still unmarried.

If the regulations of the Society admitted of a certain annual payment being made to *each* child until it attained the specified age, all that would be required of the Actuary, in estimating the liabilities under the first class, would be to find the value of a temporary annuity on each child's life. But the rule almost invariably is, that *one* annuity is payable as long as any child remains under age; and this, when there are more than one child living, renders the strict calculation of the liability a very complicated matter. A very near approximation, however, may without difficulty be attained, and the error will be on the safe side. It is evident that the longest period for which the annuity can be payable is equal to the difference between the age of the youngest child (x), and the year at which the annuity ceases (n); $(n-x)$ therefore expresses this period. Now, the value of the annuity is greater than a temporary annuity on a life aged x , because should x fail, there are other lives, y , z , &c., who may draw the annuity for a portion of the time at least. It

is also less than an annuity certain for the same period; but the deficiency is very slight in most cases, especially if the number of years which have yet to run are small, or if the family is a large one, and the members near of an age. If the value of these annuities of the first class be assumed to be certain for the period $n-x$, the approximation will be near enough for all practical purposes.

In valuing the annuities payable to the children of widows, two different contingencies require to be considered—First, That the widow shall die within a limited period; and secondly, That one or more of her children shall survive her, and live through the whole or a portion of that period. Should she have but one child, the value will be that of a limited-survivorship annuity, payable to the child aged x after her decease, provided that happens in less than n years. Supposing the mother's age to be y , the formula would then be

$$\left(A_x - A_{x+n} \cdot \frac{l_{x+n} v^n}{l_x} \right) - \left(A_{x:y} - A_{x+n:y+n} \frac{l_{x+n} \cdot l_{y+n} v^n}{l_x \cdot l_y} \right)$$

A_x representing the value of an annuity on a person aged x , and $A_{x:y}$ the value of a joint life annuity on two lives aged x and y .

But if more than one child is alive at the time of investigation, the calculation of annuities dependent on their survival becomes too complicated for ordinary practice; and it will serve all necessary ends to assume that the annuity will certainly be paid to the end of the period n years, provided the mother, aged y , dies within the term, x as before expressing the age of the youngest child, and n the difference between that age and the age at which the annuity ceases. Thus if ${}^nA^t$ represent an annuity certain for n years, the formula will be—

$${}^nA^t - \left(A_y - A_{y+n} \frac{l_{y+n} v^n}{l_y} \right)$$

If the youngest child is under one year of age, the second youngest may be taken instead; the value of life being so very small in children of this early age. In the great majority of

cases, however, it will be found extremely difficult to obtain the ages of the children, or even to ascertain which of the widows have families at all. In such instances it can only be assumed that a certain percentage have children, and that the ages of these children are proportionate to the age of their mother. Table XIX. has been calculated on the experience of the Ministers' Widows' Fund, and shows the value of an annuity to a widow's family after her death, regulated by the ages observed among those connected with that Fund.

By the regulations of many Widows' Funds, the widow's annuity ceases if she marry again; but, in the event of this happening, the children's provision becomes payable as in the case of her death. This contingency, of course, will increase the value of annuities to widows' children, but to what extent it is difficult to estimate. I do not think the risk is worth a separate calculation, more especially as by the process of calculation I have described the values are slightly overestimated.

Annuities to children of widowers, if there is little chance of their marrying again, may be valued in the same manner as those we have just now considered. But if a second marriage takes place, then, in the event of the member dying while any of his first wife's children are under age, it will not be to them that the annuity falls to be paid, but to his widow by the second marriage, if she survive him; and her children, if she should have borne him any, will draw the annuity till the youngest reaches the limiting age, should the mother die while they are still under it. Hence, in order to find a nearer approximation to the value of annuities to children of widowers, we shall require to multiply the value found by the formula applicable to widows' children by the probability that a widower of the specified age will not marry again, which will be found by Table III.; and the value thus found may be held to include what might chance to be payable in consequence of his leaving orphan children by a second marriage, as the greater difference of age in such marriages renders the survival of a widow highly probable, reducing the liability for children under such a contingency to so low an

amount that we may consider it covered by the over-estimate in other points.

The next class of annuities is payable to the children of those members who, with their wives, are both alive. When the members and their wives are both well up in years, it may be considered that no more children will be born to them; and then, proceeding on the same principle as in the other cases, the value becomes that of an annuity payable for a limited number of years after the death of the last survivor of husband and wife should they both die within the term; but when the parents are of such an age that more children may still be looked for, the calculation becomes a much more difficult one. Table XX. has been prepared to assist in such calculations. I shall afterwards explain the principle on which it is constructed.

The proportion existing between the age of husband and wife has been supposed by some to influence the number of the family, and more particularly the proportions of the sexes of the children. An endeavour was made to elucidate this point by means of the statistics of the Ministers' Widows' Fund, but the result does not seem at all conclusive. The basis, however, was by no means wide enough, and it is possible that, were much greater numbers examined, some law would be observed. Judging, however, from the facts here recorded, I would be inclined to conclude that the relative ages of husband and wife have no influence on the proportions of male and female children. The following are the figures which were collected. The first Table shows the statistics of all the marriages which had taken place among the existing members of the Fund and widows on the lists, without regard to the mother's age; and the second embraces only those cases where the father was dead, or where the mother might be considered to be past child-bearing, and therefore the family had attained its full number.

There is perhaps an indication from these figures that the number of the family is increased when the husband is older than the wife; but it will be observed that the marriages are comparatively few when the reverse is the case—too few, perhaps, to draw any conclusion from the results.

*Tables showing the Average Number of Sons and Daughters
in each Family.*

MINISTERS' WIDOWS' FUND.

HUSBAND OLDER.

	Number of Marriages.	Sons.	Daughters.	Average.	
				Sons.	Daughters.
Husband and Wife equal ages,	50	83	102	1·66	2·04
Husband one year older,	38	95	93	2·50	2·45
Husband two years older,	56	152	119	2·71	2·12
" three "	58	122	115	2·1	2·
" four "	67	141	148	2·1	2·2
" five "	63	149	144	2·36	2·29
" six "	61	143	127	2·34	2·08
" seven "	56	151	157	2·69	2·8
" eight "	66	221	192	3·35	3·
" nine "	58	165	148	2·84	2·55
" ten "	39	81	74	2·08	1·9
" eleven "	42	133	116	3·17	2·76
" twelve "	35	82	99	2·34	2·83
" thirteen "	40	95	101	2·37	2·52
" fourteen "	37	115	95	3·11	2·57
" fifteen "	25	83	70	3·32	2·8
" sixteen "	22	69	58	3·14	2·64
" seventeen "	22	54	64	2·45	2·91
" eighteen "	18	26	37	1·44	2·05
" nineteen "	24	72	64	3·	2·67
" twenty "	16	46	37	2·87	2·31
Upwards of twenty years,	61	152	146	2·49	2·39

WIFE OLDER.

Wife one year older,	36	77	70	2·14	1·94
" two "	18	39	23	2·17	1·28
" three "	25	40	29	1·6	1·16
" four "	15	23	27	1·63	1·8
" five "	6	8	5	1·33	·83
" six "	4	4	1	1·	·25
" seven "	5	9	6	1·8	1·2
" eight "	2	1	2	·5	1·
" nine "	3	2	2	·66	·66
" ten "	4	...	2	...	·5
Upwards of ten years older,	2	2	...	1·	...

Tables showing the Average Number of Sons and Daughters in each Family where the Mother was aged 45 years and upwards.

HUSBAND OLDER.

	Number of Marriages.	Sons.	Daughters.	Average.	
				Sons.	Daughters.
Husband and Wife equal ages,	37	59	66	1·6	1·78
Husband one year older, .	31	71	78	2·29	2·52
" two " .	28	99	75	3·54	2·68
" three " .	32	88	84	2·75	2·62
" four " .	43	99	108	2·3	2·51
" five " .	35	102	94	2·91	2·69
" six " .	30	102	74	3·4	2·47
" seven " .	32	110	118	3·44	3·53
" eight " .	44	145	144	3·3	3·27
" nine " .	25	98	76	3·84	3·04
" ten " .	20	42	39	2·1	1·95
" eleven " .	24	94	80	3·91	3·33
" twelve " .	18	47	44	2·61	2·44
" thirteen " .	17	45	48	2·65	2·82
" fourteen " .	18	70	57	3·89	3·17
" fifteen " .	15	59	44	3·93	2·93
" sixteen " .	14	48	32	3·43	2·29
" seventeen " .	11	33	36	3·	3·27
" eighteen " .	3	2	5	·67	1·67
" nineteen " .	10	30	29	3·	2·9
" twenty " .	8	23	23	2·88	2·88
Upwards of twenty years older,	21	77	70	3·67	3·33

WIFE OLDER.

Wife one year older, . .	24	61	52	2·54	2·17
" two " . .	11	25	16	2·27	1·45
" three " . .	19	32	21	1·68	1·1
" four " . .	8	13	17	1·62	2·13
" five " . .	5	8	5	1·6	1·
" six " . .	4	3	1	·75	·25
" seven " . .	4	8	5	2·	1·25
" eight " . .	1
" nine " . .	2	2	1	1·	·5
" ten " . .	4	...	2	...	·5
Upwards of ten years older, .	1	2	...	2·	...

The preceding Tables show the original number of sons and daughters in each family ; and in arranging the statistics every family was included, those in which there had been no children not being excepted.

The average number of children in each family is somewhat remarkable, and this high average does not appear to be much influenced by the age of the father at marriage, for it will be seen that the average number of children in each family is no greater when the husband is but little older than his wife than in those cases where there is a much wider difference between the ages.

The average number of children in those families where the husband was not more than five years older than his wife at marriage is 5·03, and the fathers of these families, we may presume, had married comparatively early in life. But in those cases where, from the greater disproportion between the age of husband and wife, it is to be supposed that marriage had been delayed, we find little difference in the number of the family, the average among those where the wife was upwards of fifteen years younger than her husband at marriage being 4·98.

No method of forming an estimate of the value of the last class of children's annuities can be pointed out which does not involve much uncertainty, and far more labour than the object is worth, for the present value of annuities to the children of members still unmarried must be so very small, that it may fairly be left to the Actuary to assume an amount which he may consider sufficient to cover the risk.

Instead of paying annuities to the orphan children, it is fully as common to provide for a single payment to be made at the death of the last surviving parent.

This is generally paid irrespective of the age of the surviving family. Table XXI. has been calculated to assist in making such valuations in the cases where both parents are still alive. When one parent only survives, the liability will be simply the value of a reversion payable at his or her death ; and if it is not known which of the surviving parents have families, each case

must be valued, and from the total a deduction must be made of the estimated percentage having no children.

The only matter now remaining to be considered, is the question of expenses.

It is difficult to lay down any rule for valuing this item, for the expenses of management vary so much in different Societies, that the past history of each case must be studied in order to arrive at any idea on the point. It may, however, be reasonably supposed that whatever the Society's prosperity may be, the future expenses will bear some proportion to the annual income, and therefore in many cases it may answer very well to set aside a percentage upon the value of the future annual rates, the percentage to be determined by the proportion which the expenses have hitherto borne to the income from this source.

In other cases, where a rate of interest has been adopted as the basis of the valuation lower than has been realised on the past investments of the Society, and lower also than may be looked for in future on the class of securities allowed by its regulations, it may not be necessary to set aside a sum to cover expenses, particularly if the Society is economically conducted, for the expenses may be considered as provided for by the additional interest to be received.

All the points usually requiring to be considered in the valuation of Widows' Funds have now been indicated, and I have endeavoured, as far as possible, to be practical in pointing out the way in which the values can readily be ascertained. The great object of the Actuary should be to secure accuracy, but, at the same time, it is necessary for him to endeavour to save labour. So many different combinations will be found in the ages of the various lives connected with a fund of even limited extent, that to value each case minutely, and to enter into a calculation of all the different reversionary and survivorship contingencies arising out of each family, would involve an amount of labour which no Actuary would undertake and no Society would pay for. But by adopting an average age for husband and wife in calculating the risks attendant on second marriages, and

by following some such process as I have suggested in valuing children's payments, a very close approximation to the actual values will be obtained. And, as it will frequently be found, no information can be obtained as to the ages of the children, for the records of these Societies are often very imperfect, and there is frequently a disinclination to issue schedules for the purpose of collecting facts to assist in the calculation, the Actuary, under such circumstances, has no resource but to proceed upon the experience of other Societies, and to employ Tables, such as those which are supplied in this work.

DESCRIPTION OF THE TABLES.

In the year 1834 the late Mr Cleghorn published a small volume on the subject of Widows' Funds, at the end of which he printed a Table of first marriages, computed from the experience of the Ministers' Widows' Fund. I quote Mr Cleghorn's account of this Table. In page 29 of his book he says, "From two lists of members of 100 each, taken from the books of the Ministers' Fund as they entered, without omission or selection, the one beginning in 1749 and ending in 1753, and the other beginning in 1760 and ending in 1763, it appears that the number who married out of the former was 82, and out of the latter 85, of which the mean for these 200 members is 83·5 per cent. . . . In the Ministers' lists, 25 or 12·5 per cent. were married at admission; but, assuming the average age of the whole to be then 28, and that half of these 12·5 married at ages not exceeding 30, and the other half at ages not exceeding 35, and combining the two lists, the ages at which the 83·5 first marriages of 100 members were contracted, will be as follows, viz.—

Not exceeding 30,	.	.	.	38·25
" " 35,	.	.	.	27·75
" " 40,	.	.	.	10·
" " 50,	.	.	.	4·
" " 60,	.	.	.	3·
" " 62,	.	.	.	·5
				Sum, 83·5."

On page 75 Mr Cleghorn again says—"The years of admission and of marriages for those who were unmarried at admission, are

to be found in the lists referred to, and, consequently, if the usual age at admission be 28, the ages at their marriage will be as in column 1. The marriages in column 3 are accordingly ascribed to the ages in column 1 upon this assumption. But those who were married at admission, and for whom the year of marriage is not given, are distributed among the ages from 24 to 35."

The following is the Table which he constructed :—

MR CLEGHORN'S TABLE.

AGE.	Number who complete that Age.	Number who Marry.	Number of Bachelors in Col. 2.	Probability of Marriage.
24	100	...	100	8350
25	99.29	1	98	8315
26	98.56	3	94	8277
27	97.84	3	91	8235
28	97.08	9	81.5	8188
29	96.23	10	70.67	8136
30	95.29	12.25	57.79	7830
31	94.33	6.5	50.74	7637
32	93.36	5.25	45	7444
33	92.42	6	38.57	7130
34	91.49	5	33.21	6775
35	90.56	5	27.90	6272
36	89.63	2	25.62	6050
37	88.68	3.5	21.87	5487
38	87.72	2	19.64	5092
39	86.74	.5	18.92	5021
40	85.71	2	16.71	4488
41	84.60	.5	16	4375
42	83.43	.5	15.28	4254
43	82.23	.5	14.56	4121
44	81.03	*	*	*
45	79.83	*	*	*
46	78.65	.5	13.43	4095
47	77.49	.5	12.74	3925
48	76.36	1	11.56	3460
49	75.29	.	.	.
50	74.26	.5	10.75	3256
51	73.27	.5	10.11	2967
52	72.22	.5	9.47	2640
53	71.12	1.5	7.84	1276
.	.	.	*	.
60	61.53	1	5.8	...

The asterisk (*) denotes that there were no marriages in the lists referred to at these ages, or between the preceding and following ages.

It will be seen from the above extract that Mr Cleghorn had not sufficient data from which to construct this Table. He was not in possession of the actual age of the members whose marriage he dealt with, for the *ages* of its members were never recorded in

the books of the Ministers' Widows' Fund. He took, it will be noticed, the first 100 members who entered after 1749, and the same number who first joined the Fund after 1760, and observing the dates of their marriages as they appeared in the subsequent Annual Reports, he *assumed* that the age at entry in each case was 28, and, proceeding upon this assumption, he estimated the ages at which the marriages took place to be 28, plus the number of years which had elapsed since their admission. Mr Cleghorn's Table, then, was almost entirely founded upon an assumption, his only ascertained facts being the total percentage of marriages, and the number of years between the member's admission and the date of his marriage. This Table, however, was for many years the only authority upon the value of future marriages, and upon it the late Mr Griffith Davies proceeded in framing the manuscript Tables before referred to. Being in possession not only of the dates of marriage, but also of the actual ages of the members and their wives whose marriages are recorded, these having been ascertained, as I have already stated, by the issue of schedules to each individual member, I have had greatly the advantage of Mr Cleghorn, and have been obliged to assume nothing, but have proceeded entirely upon facts. The number of first marriages observed was 1440 among the ministers, and 885 among the schoolmasters.

The first process in the preparation of the Tables of first marriages was to record carefully the age attained, in years and months, of all the existing members, and also the exact ages at death of the members who had died in the previous eighteen years, and whose ages, as already stated, had been ascertained. The exact ages at first marriage of the living members were obtained from the schedules; the date of the first marriage of the deceased members having been found in the Annual Report, and their age at death being known, it was easy to calculate their age at marriage. The ages at which the first marriages occurred which took place among the deceased members before their admission to the Fund, were not ascertained; but the member's age at admission being known, his age at marriage was assumed to be

the same as was found to be the average among the other members who married before the age at which he was admitted.

Proceeding, then, with these facts, and following the same method as is ordinarily pursued in framing a Table of Mortality, I found how many members lived *through* each year of age in a condition of bachelorhood. I then assumed that the marriages which were found to have taken place in each year of age were distributed at equal intervals throughout the year, and that consequently the number of bachelors living through the year was a mean proportion between those who were unmarried at the beginning of the year and those who remained bachelors at the end.

The following Table shows the percentage thus obtained:—

Age.	Number Marrying out of 100 living through the year.	Age.	Number Marrying out of 100 living through the year.	Age.	Number Marrying out of 100 living through the year.	Age.	Number Marrying out of 100 living through the year.	Age.	Number Marrying out of 100 living through the year.
20	·1234	30	6·5269	40	7·3786	50	5·1846	60	1·5755
21	·2423	31	6·5044	41	6·8511	51	3·3469	61	1·5139
22	·4068	32	8·1610	42	5·8822	52	3·4933	62	1·3465
23	1·1385	33	6·7238	43	4·9146	53	2·8708	63	1·2634
24	1·5746	34	7·6007	44	3·4582	54	2·4741	64	1·0406
25	2·4417	35	8·7531	45	4·8724	55	2·6172	65	·8734
26	3·8654	36	6·6136	46	4·0158	56	2·2535	66	·6993
27	4·9102	37	7·0574	47	3·9078	57	2·2086	67	·6394
28	5·2383	38	9·1823	48	5·4816	58	1·9092	68	·5579
29	6·7985	39	8·1129	49	3·5932	59	1·6197	69	·4525

As the figures in the above Table represent the number who marry out of 100 bachelors living through each year, it was necessary, before constructing a Table of Marriages, to find the proportion which married out of 100 beginning the year. The Carlisle Table of Mortality being adopted, it is evident by the formula $\frac{1}{2}(l_x + l_{x+1}) : l_x :: 100 : 100 + c$, that $100 + c$ will represent the number beginning the year x , of whom 100 live through it. If then p_x be assumed to represent the percentage in the above Table at age x , it follows the $100 + c : 100 :: p_x : p'_x$, where

p'_x represents the number who marry out of 100 beginning the year.

Having assumed that the bachelors would die off according to the Carlisle rate of Mortality, the first and second Tables, showing how many bachelors will commence each year of age out of 2000 who are unmarried at the age of 20, according to the marriage laws observed among the members of the Ministers' and Schoolmasters' Widows' Funds, were prepared. The following Table will explain the process adopted in their construction:—

Age.	Number Marrying out of 100 beginning the year.	Bachelors attaining each age.	Number Married in each year.	Bachelors, less the number who Marry.	Deaths among Bachelors in previous Column.
20	·1230	2000·	2·5	1997·5	14·1
21	·2414	1983·4	4·8	1978·6	13·7
22	·4054	1964·9	8·	1956·9	13·7
23	1·1345	1943·2	22·	1921·2	13·5
24	1·5690	1907·7	29·9	1877·8	13·3
25	2·4328	1864·5	45·4	1819·1	13·3
26	3·8512	1805·8	69·5	1736·3	12·8
27	4·8911	1723·5	84·3	1639·2	12·7
•	• • •	• • •	• •	• • •	• •

The probability of marriage was found by dividing the number of marriages at and above every age by the number of bachelors entering upon that year.

For if b_x represent the number of bachelors attaining any year x , and m_x represent the number who marry in the following year, the probability of marriage in that year will be $\frac{m_x}{b_x}$, and the probability of being still unmarried at the end of the year will be $\frac{b_{x+1}}{b_x}$. The probability of marrying in the second year will be $\frac{m_{x+1}}{b_{x+1}} \times \frac{b_{x+1}}{b_x}$ or $\frac{m_{x+1}}{b_x}$. In like manner, it can be shown that the probability of marriage in the third year is $\frac{m_{x+2}}{b_x}$, and that the probability of marriage at age $x + n$

will be $\frac{m_{x+n}}{b_x}$. The sum of the probability of the marriage of a bachelor, aged x , will be

$$\frac{m_x + m_{x+1} + m_{x+2} + m_{x+3} + \dots + m_{x+n}}{b_x}$$

$x + n$ representing the age at which the last marriage takes place, as shown by the Table.

The age of each individual who marries at age x is, of course, represented by x , and the sum of the ages of all who marry in this year will be $x \cdot m_x$. The sum of the ages of all who marry in the succeeding year will be $(x + 1) \cdot m_{x+1}$. The average age, therefore, of those who marry in those two years will be

$$\frac{x \cdot m_x + (x + 1) \cdot m_{x+1}}{m_x + m_{x+1}}.$$

In the same manner it can be shown that the average age at marriage of the bachelors who attain the age of x is

$$\frac{x \cdot m_x + (x + 1) \cdot m_{x+1} + (x + 2) \cdot m_{x+2} + \dots + (x + n) \cdot m_{x+n}}{m_x + m_{x+1} + m_{x+2} + m_{x+3} + \dots + m_{x+n}}$$

$x + n$, as before, representing the age at which the last marriages are shown by the Table.

The average age at marriage in Tables I. and II. was, therefore, found by taking the sum of the ages of all who marry at each age and upwards, and dividing it by the total number of marriages.

The results shown by the first and second Tables are somewhat different. The schoolmasters get their appointments at an earlier age. The average age at entry in their society is 28·7, while in the Ministers' Widows' Fund it is 32·5. Marriages among the schoolmasters therefore take place sooner, and also in somewhat greater number than among the ministers. One-half of the schoolmasters are married at the age of 30, but the half of the ministers are not married until three years later. The probability of a bachelor marrying at all is therefore greater by the first

Table previous to the age of 25, but after this age there is a greater probability of marriage shown by the second Table, owing to the marriages having been delayed; and, consequently, a larger number of bachelors who will yet marry will now be found among the ministers. The average ages of the wives taken by those who married at each age were found by actual observation. These, it will be seen, are nearly the same at each age in both Tables; but owing to the larger proportion of early marriages among the schoolmasters, and the tendency of very young husbands to take wives as old as themselves, or even older, the difference between the average age of husband and wife at marriage is less in the one case than in the other, the ages being—

In Schoolmasters' Widows' Fund—Husbands,	30·6
„ Wives,	25·6
In Ministers' Widows' Fund—Husbands,	33·4
„ Wives,	26·1

The differences between the ages of husband and wife in the Ministers' Widows' Fund will probably be found more nearly approaching the general experience of those societies whose members are drawn from the higher classes of society. At the last investigation of the Widows' Fund of the Faculty of Advocates in Scotland, the average age of husband and wife at first marriage was stated to have been 31·2 and 24·8, while ten years difference was stated to have been observed between the ages of husband and wife in the Society of Writers to the Signet in Scotland at a late investigation. This difference, however, was excessive, and must have arisen from exceptional causes.

TABLE III.—*Probability of the Re-marriage of Widowers.*

In forming a Table of the law of marriage among widowers, it is at once apparent that a difficulty presents itself which has no place in the calculations relating to men who have not yet married. In the latter case, there is nothing which to an Actuary can be held as a reason for putting a difference between one man and another, or supposing that there should be any variation in

the probability of marriage of individuals of the same age. But when we come to consider the probability of the re-marriage of widowers, the idea suggests itself that the probability of re-marriage at any particular age may possibly depend in some measure upon the length of time which has elapsed since the first wife died. At age 50, for instance, it is quite possible that the probability of the re-marriage of a widower whose wife has been dead for 20 years may be greater or less than that of another whose wife has been dead only 10 years. To judge satisfactorily, therefore, of the probable law of second marriage among widowers of any age, x , we should proceed only upon the observation of those who were widowers at this age, and not only so, but we should require to separate all the widowers at each age according to the period of their widowerhood. It would be impossible to collect statistics sufficient for such an arrangement, and in order to be in a position to arrive at any conclusions at all in regard to the re-marriage of widowers, we must, I think, leave out of the calculation the possibility that the previous duration of widowerhood has any influence upon the probability of re-marriage, and we must be content to observe the law of marriage among widowers of all ages, without reference to the date of their wives' death.

There were in all 225 second marriages recorded among the members of the Ministers' and Schoolmasters' Widows' Funds. By arranging these marriages, therefore, according to age, and observing what proportion they bore to the total number of widowers who lived *through* each year of life, a scale of second marriages was obtained, and from this the Table of the probability of re-marriages was calculated; but as the method adopted to obtain these results was precisely similar to the course followed in ascertaining the probabilities of first marriage, already described, it is unnecessary to repeat the explanation.

At each age in this Table is given the average age of the widower's second wife, as observed in the two Funds, taking, however, the average of a series of years.

TABLES VI. AND VII.—*Value of Annuity of £1 to Widows of Ministers and Schoolmasters, still Unmarried.*

At the date of the marriage of an individual aged x , the value of an annuity to his widow will be $A_y - A_{x \cdot y}$, where y represents the age of his wife. If the marriages which take place at age x , are assumed to occur at equal intervals throughout the year, the ages at marriage will be $x \cdot y$, $x' \cdot y$, $x'' \cdot y$, $x''' \cdot y$, &c., and the average age $(x + \frac{1}{2}) \cdot y$. The value of an annuity, payable to the widow of an individual who marries at age $x + \frac{1}{2}$, will therefore be $A_y - A_{x+\frac{1}{2} \cdot y}$, and the value of an annuity to the widow of an individual who marries in the middle of the following year, will be $A_{y'} - A_{x+1+\frac{1}{2} \cdot y'}$, and so on for every succeeding year.

If now the number of bachelors at age x be represented by b_x , and the number who marry in the following year by m_x , the average value at the instant of marriage of a survivorship annuity, payable to the widow of any one of them, will be

$(A_y - A_{x+\frac{1}{2} \cdot y}) \frac{m_x}{b_x}$, and this must be discounted for six months

in order to obtain the value at the beginning of the year, or at the precise age of x . The value at age x will therefore be

$$(A_y - A_{x+\frac{1}{2} \cdot y}) \frac{m_x}{b_x} \times \frac{1}{(1+r)^{\frac{1}{2}}}.$$

At age x , the average value of the annuity to the widows of those who marry in the second year will be

$$(A_{y'} - A_{x+1+\frac{1}{2} \cdot y'}) \frac{m_{x+1}}{b_x} \times \frac{1}{(1+r)^{\frac{1}{2}}} \cdot \frac{1}{(1+r)};$$

and the average value of the annuity, payable to the widows of those who marry in the third year, will be

$$(A_{y''} - A_{x+2+\frac{1}{2} \cdot y''}) \frac{m_{x+2}}{b_x} + \frac{1}{(1+r)^{\frac{1}{2}}} \cdot \frac{1}{(1+r)^2},$$

and so on for every succeeding year as long as there is any probability of marriage shown by the Table.

If, then, we represent the successive survivorship annuities by A' , A'' , A''' , A'''' , &c., and the value of £1 due 1, 2, 3, 4, &c. years

hence, by v^1, v^2, v^3, v^4 , &c., the sum of the series at age x becomes

$$(A'm_x + A''m_{x+1}v' + A'''m_{x+2}v'' + A''''m_{x+3}v''' + \&c.) \frac{v^4}{b_x},$$

and at age $x + 1$ the sum of the series will be

$$(A''m_{x+1}v^1 + A'''m_{x+2}v^2 + A''''m_{x+3}v^3 + A'''''m_{x+4}v^4 + \&c.) \frac{v^{\frac{5}{2}}}{b_{x+1}},$$

and so on for every succeeding year; and if, as in calculating the two preceding Tables, the elements which compose these quantities be arranged in parallel perpendicular columns, the desired results are ascertained, as may be seen by an examination of the following Table :—

AGE.		Surv. Annuities, Husband Six Months Older.	Previous Column discounted Six Months.	Value of £1, due 0, 1, 2, 3, &c. years hence.	No. Marrying in each year.	Product of three preceding Columns.	Additions of preceding Column.	No. attaining each Age unmarried.	Preceding Column multiplied by value of £1, due 0, 1, 2, 3, &c. years hence.	Value of Annuity, 3 per cent.
H.	W.									
20	20	3.751	3.696	1.000000	2.5	9.0889	4726.9968	2000.	2000.	2.3635
21	21	3.760	3.704	.970874	4.8	17.2231	4717.9079	1983.4	1925.6	2.4501
22	21	3.869	3.812	.942596	8.0	28.6193	4700.6848	1964.9	1852.1	2.5380
23	22	3.884	3.827	.915142	22.0	77.2004	4672.0655	1943.2	1778.3	2.6273
24	23	3.900	3.842	.888487	29.9	102.1850	4594.8651	1907.7	1695.0	2.7109
*	*	***	***	***	**	***	***	***	***	***

The first two columns contain the ages of husband and wife at the beginning of the year of marriage. The third column is a series of survivorship annuities, in which the male life is six months older, the female being of the Table age. In order to obtain the immediate value of these annuities, they are discounted for six months, and the values thus found form the fourth column. The fifth column is the value of £1 due immediately, and discounted for 1, 2, 3, 4, &c. years. The numbers marrying in each year are found in the next column, and these three last columns being multiplied together make up the seventh column. The eighth column is formed by the summation of the numbers in the seventh column at each age and at all higher ages, and this being divided by the product of the members attaining each age unmarried and the value of £1 due im-

mediately, and discounted for 1, 2, 3, 4, &c. years, produces the last column, or the value of an annuity of £1 to the widows who may be left through the marriage of males still unmarried.

TABLE VIII.—*Value of an Annuity of £1 to the Widows who may be left through the Re-marriage of Widowers.*

The principle upon which this Table is formed is precisely similar to that followed in the calculation of those immediately preceding. It will not be necessary, therefore, that I should again enter fully into an explanation of that principle, but I subjoin a few lines of the columns used in its preparation, which will enable me to explain its construction.

AGE.		Value discounted Six Months of a Survivorship Annuity, Husband half year older.	Widowers Marrying in each year.	Value of £1 due any number of years hence.	Product of the three last Columns.	Summations of the preceding Column.	Product of number of Widowers attaining each age, and the value of £1 due any number of years hence.	Value of Annuity, 3 per cent.
H.	W.							
28	27	3·8966	19·72	1·000000	76·8348	680·1671	195·55	3·4782
29	27	4·0039	24·04	·970874	93·4566	603·3323	169·22	3·5653
30	27	4·1084	19·20	·942596	74·3427	509·8757	140·24	3·6357
31	27	4·2167	17·15	·915142	66·1810	435·5330	117·39	3·7103
32	27	4·3340	16·21	·888487	62·4374	369·3520	97·73	3·7795
		* * *	* *	* * *	* * *	* * *	* *	* *

The first two columns contain the age of the widowers and of their second wives. The next column contains the value discounted for six months of a survivorship annuity, in which the male life is half a year older than the husband's age in the first column, as it is presumed that the re-marriages which take place in each year will, one with another, be six months deferred. The fourth column contains the number of widowers who marry in each year, and the fifth column contains the value of £1 due 0, 1, 2, 3, 4, &c. years hence. Column six shows the product of the three preceding columns, and column seven is obtained by summing together the numbers in column six. The eighth column is the product of the number of widowers attaining

each year of age, and the value of £1 discounted for 0, 1, 2, 3, &c. years, and by dividing the numbers in the seventh column by the corresponding numbers in the eighth, the value of £1 annuity to the widows who may be left through the re-marriage of widowers is obtained.

TABLES IX. TO XVI.—*Value of an Annuity to the Widows of the Second Marriages of Males now living in Wedlock.*

If at the commencement of any year we assume that there are a number of married pairs corresponding to the number living at the same age in the Carlisle Table of Mortality, and if the age of the husband be represented by x , and that of the wife by y , then the number of pairs who live to the end of the year must be equal to the number in the Carlisle Table at age $x + 1$, multiplied by the probability that a life aged y will also survive the year,—that is, the pairs living at the end of the year will be represented by $l_{x+1} \cdot \frac{l_y+1}{l_y}$.

The number of pairs living at the close of the following year must also be equal to the numbers living at age $x + 2$, multiplied by the probability that a life aged y will survive two years, that is, $l_{x+2} \cdot \frac{l_y+2}{l_y}$.

The pairs living at the end of the third year can also be shown to be equal to $l_{x+3} \cdot \frac{l_y+3}{l_y}$, and so on for each succeeding year, the number of husbands and wives surviving n years being represented by $l_{x+n} \cdot \frac{l_y+n}{l_y}$.

Now, if from the total number of males surviving one year, l_{x+1} , we deduct the husbands living, namely, $l_{x+1} \cdot \frac{l_y+1}{l_y}$, we obtain the number who become widowers in the first year, and live to the end of it.

To ascertain the number who become widowers in the second year, and live to the end of it, we must first discover how many of the males who are married at the beginning of the year will survive. This number will be represented by $\frac{l_{x+1} \cdot l_{y+1}}{l_y} \times \frac{l_{x+2}}{l_{x+1}}$; and if from this we take the num-

ber of pairs surviving, as shown above, to be $\frac{l_{x+2} \cdot l_{y+2}}{l_y}$, we obtain

the number of widowers in the second year, namely,

$$\frac{l_{x+1} \cdot l_{y+1}}{l_y} \times \frac{l_{x+2}}{l_{x+1}} - \frac{l_{x+2} \cdot l_{y+2}}{l_y}.$$

And, generally, to obtain the widowers in each year we must multiply the number of pairs commencing the year by the probability of the male life surviving, and from the product deduct the number of pairs commencing the year following, and the number who become widowers in the n^{th} year will then be

$$\frac{l_{x+n-1} \cdot l_{y+n-1}}{l_y} \times \frac{l_{x+n}}{l_{x+n-1}} - \frac{l_{x+n} \cdot l_{y+n}}{l_y}.$$

An inspection of the first seven columns of the following Table will show that the number of the widowers upon which the values of annuities payable are based was calculated according to the above formulæ.

AGE.		Log. probability of a Life aged 1, 2, 3, 4, &c. years.	Previous Column + Log. living at each age after 24.	Pairs Living.	Males living one year.	Widowers in next year.	Product of previous Column, and Annuity to Widows discounted.	Summation of Numbers in previous Column.	Value of Annuity, 3 per cent.
H.	W.								
24	17	5921	...	40-650	137-272	3266-665	55171
25	18	1-9969867	3-7662902	5838-350	5879	40-352	132-297	3129-393	55209
26	19	9939524	7600677	5755-296	5795-648	40-056	127-501	2997-096	55247
27	20	9908967	7538002	5672-835	5712-891	39-744	122-824	2869-595	55276
28	21	9878194	7473362	5589-026	5628-770	38-483	118-353	2746-771	55314
		* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *

The first two columns are the assumed ages of husband and wife. Commencing, then, with a number of married pairs, aged

24 and 17, corresponding to the number living at age 24 in the Carlisle Table, namely 5921, it is evident that the number of pairs surviving each succeeding year will be equal to the number of males attaining each age, according to the Carlisle Table, multiplied by the probability of a life of 17 surviving the same successive periods. Column third shows the logarithm of the probability of a life of 17 surviving 1, 2, 3, 4, &c. years. The fourth column gives the sum of the logarithms in the previous column, and the logarithms of the numbers attaining each age after 24. Column five contains the numbers corresponding to column four, or the pairs living at each year of age. In column six are found the number of males in column five who live to the end of one year. The difference, therefore, between these two columns must be the number who become widowers in each year and live to the end of it.

Now, if the wives of those men may be assumed to live on an average half through the year in which they die, it may be taken for granted that no widower will marry again before other six months have elapsed, and that, consequently, those widowers who die in the same year with their wives will in no instance have married again.

Taking into account, therefore, only those who live to the end of the year in which they become widowers, and representing the widowers at age x , and in the following years by

$$w_x \cdot w_{x+1} \cdot w_{x+2} \cdot w_{x+3}, \text{ \&c. ;}$$

and the values of the annuities in Table VIII. by

$${}^wA_x \cdot {}^wA_{x+1} \cdot {}^wA_{x+2} \cdot {}^wA_{x+3}, \text{ \&c.,}$$

we find that the total values of the annuities payable to the widows who may be left through the second marriage of those who become widowers in the first year after age x , will, at the time of their wives' death, be ${}^wA_x \times w_x$; and if this be discounted for six months, the total values at the beginning of the year x are found.

But if the number of pairs living at age x be represented by p_x , the value of an annuity to the widow who may be left through

the second marriage of any one of those who become widowers in the first year will be expressed by

$${}^wA_x \cdot \frac{w^x}{p_x} \times \frac{1}{(1+r)^{\frac{1}{2}}}.$$

The value of an annuity to the widow who may be left through the second marriage of a married man aged x , who may become a widower in the second year, may also be shown to be expressed by

$${}^wA_{x+1} \frac{w_{x+1}}{p_x} \times \frac{1}{(1+r)^{\frac{3}{2}}},$$

and so on for the following years.

On this principle the Table in question was formed.

The eighth column, given above, shows the product of the discounted value of the annuities payable to the widows who may be left through the re-marriage of widowers (Table VIII.) and the number of widowers in each succeeding year. Column nine contains the summation of the successive numbers in the previous column, and by dividing them by the product of the pairs living at each age, and the value of £1 due any number of years hence, the required values of £1 annuity are obtained.

TABLE XVII.—*Value of £1 Annuity to the Widows who may be left through THE SECOND MARRIAGES of Bachelors.*

This Table is formed from the preceding, as shown in the following specimen of the columns used in its construction; for if the bachelors who marry in any year x are supposed to marry at equal intervals through the year, the average age at marriage will be $x + \frac{1}{2}$, and therefore the value of an annuity to the widows who may be left through their second marriage may, at the date of the first marriage, be assumed to be a mean proportion between the values at age x and at age $x + 1$ in the Table of the value of an annuity of £1 to the widows left through the second marriage of males living in wedlock.

Age.	Value of £1 Annuity to Widows of Married Pairs at age x and $x + 1$.	Previous Column divided by 2.	Product of previous Column and number married in each year.	Last Column discounted for 0, 1, 2, 3, &c. years.	Sum of the Numbers in last Column.	Value of £1 Annuity, 3 per cent.
20	1·10380	·55190	1·337	1·337	517·178	·25859
21	1·10380	·55190	2·604	2·528	515·841	·26788
22	1·10380	·55190	4·332	4·082	513·313	·27715
23	1·10380	·55190	11·988	10·970	509·231	·28635
24	1·10380	·55190	16·277	14·460	498·261	·29397
25	1·10456	·55228	24·683	21·290	483·801	·30081
26	1·10523	·55262	37·868	31·710	462·511	·30583
• •	• • •	• • •	• • •	• • •	• • •	• • •

The first column shows the sum of the values of an annuity of £1 to the widows of married pairs at each age and at the age immediately succeeding, the difference in the ages of husband and wife being seven years. The second column gives one-half the sum of these values. The third column shows the value of the numbers now found multiplied into the number of ministers who marry in each year, and also discounted for six months ; and the succeeding column is the product of the numbers in the third column multiplied by the value of £1, due 0, 1, 2, 3, 4, &c. years hence. In the fifth column are found the summations of the numbers in the fourth column ; and these summations being divided by the product of the numbers attaining each age unmarried, and the value of £1 due 0, 1, 2, 3, 4, &c. years hence, give the values of the annuities payable to the widows of the *second* marriages of men who are yet unmarried according to the experience before mentioned.

PAYMENTS TO CHILDREN.

It was ascertained from the statistics of the Widows' Funds of the Ministers and Schoolmasters that the percentages of mothers living at each period of life and having families, some of whom were under 21, was on an average as follows :—

18 to 24 inclusive,	57.1
24 „ 34 „	82.4
34 „ 44 „	85.2
44 „ 54 „	80.
54 „ 59 „	61.5
59 „ 64 „	28.6
64 and upwards,	5.2

The method employed to ascertain the number of widowers surviving each year has been already described. The number of widows surviving each year was obtained in a similar way ; and by subtracting the number of widowers and widows surviving each year from the number of pairs dropped during the year, the number of marriages dissolved by the death of both parents was obtained. The following Table was thus constructed, and upon it was based the valuation of the annuities and payments to children :—

Husband's Age.	Wife's Age.	Pairs alive at beginning of each year.	Widowers in next year.	Widows in next year.	Both Parents dead next year.
27	20	5672.8	39.7	43.8	.3
28	21	5589.0	38.5	43.3	.3
29	22	5501.9	38.1	43.6	.4
30	23	5409.8	37.7	44.4	.3
31	24	5317.4	37.3	44.0	.3
32	25	5225.8	37.8	42.5	.5
33	26	5135.0	37.4	41.3	.4
34	27	5045.9	38.8	40.8	.4
35	28	4955.9	42.7	40.4	.4
36	29	4862.4	47.3	40.8	.5
37	30	4763.8	47.6	41.2	.5
38	31	4664.5	47.1	41.6	.5

Table of Annuities and Payments to Children—continued.

Husband's Age.	Wife's Age.	Pairs alive at beginning of each year.	Widowers in next year.	Widows in next year.	Both Parents dead next year.
39	32	4565.3	45.7	53.7	.5
40	33	4465.4	44.3	57.5	.6
41	34	4363.0	43.7	59.5	.6
42	35	4259.2	43.1	60.5	.6
43	36	4155.0	43.2	59.9	.7
44	37	4051.2	43.3	59.3	.7
45	38	3947.9	43.4	57.8	.7
46	39	3846.0	45.0	56.3	.7
47	40	3744.0	48.0	54.0	.6
48	41	3641.4	49.5	50.0	.7
49	42	3541.2	50.2	47.8	.7
50	43	3442.5	49.5	45.5	.7
51	44	3346.8	48.8	47.2	.7
52	45	3250.1	47.4	48.7	.7
53	46	3153.3	46.0	50.2	.7
54	47	3056.4	43.9	50.9	.7
55	48	2960.9	40.5	52.3	.8
56	49	2867.3	38.5	53.8	.6
57	50	2774.4	36.4	57.2	.9
58	51	2679.9	37.4	63.8	1.0
59	52	2577.7	38.1	71.8	1.1
60	53	2466.7	38.5	81.3	1.3
61	54	2345.6	38.2	82.6	1.3
62	55	2223.5	38.4	81.7	1.5
63	56	2101.9	38.4	78.9	1.5
64	57	1983.1	39.8	77.2	1.6
65	58	1864.5	43.3	74.7	1.9
66	59	1744.6	47.2	72.0	2.2
67	60	1623.2	51.9	69.7	2.4
68	61	1499.3	51.2	67.2	2.4
69	62	1378.4	49.0	65.2	2.5
70	63	1261.7	45.8	62.6	2.5
71	64	1150.8	43.1	65.0	2.7
72	65	1040.0	39.8	67.9	3.0
73	66	929.3	36.4	69.5	3.1
74	67	820.3	33.1	70.7	3.3
75	68	713.2	30.0	65.0	3.1
76	69	615.1	27.1	60.2	3.1
77	70	524.7	24.2	53.4	3.0
78	71	444.1	23.3	45.5	2.8
79	72	372.5	22.4	41.1	3.0
80	73	306.0	21.0	34.3	2.9
81	74	247.8	19.3	30.1	3.1
82	75	195.3	16.0	24.9	2.6
83	76	151.8	13.3	20.5	2.4
84	77	115.6	10.4	16.4	2.0
85	78	86.8	7.8	13.5	1.7
86	79	63.8	6.1	10.2	1.5
87	80	45.4	4.3	8.7	1.2
88	81	31.2	3.3	5.9	.9
89	82	21.1	2.3	3.9	.7
90	83	14.2	1.6	3.1	.6
91	84	8.9	1.0	2.1	.4
92	85	5.4	.7	1.2	.3
93	86	3.2	.5	.6	.2
94	87	1.9	.3	.4	.1
95	88	1.1	.2	.2	.0
96	89	.7	.1	.1	.1
97	90	.4	.1	.1	
98	91	.2	.0	.1	
99	92	.1	.1		
100	93	.0			

TABLE XVIII.—*Annuity after the Death of a Widower, and Payable until the Youngest Child attains the Age of 21.*

It was assumed in these calculations that when a family survived, any of the members of which were under 21 years of age, the annuity would certainly be paid for the entire term. The first column, therefore, employed in the construction of the Table was the value of an annuity certain from the youngest child's age until 21, the average age of the youngest child having been calculated from the experience of the Ministers' Widows' Fund. Subtracting from the numbers in this column the values of a temporary annuity on the father's life, we obtain the value of an annuity payable after the father's death until his youngest child reaches 21.

The values in the next Table were obtained in a similar manner; and these two Tables having been calculated, the next step was to ascertain the values of the annuities payable to the surviving children after the death of both parents.

TABLE XX.—*Value of an Annuity of £1 after the Death of both Parents, and Payable until the Youngest Child reaches 21.*

Two calculations required to be made to obtain this result. First, to ascertain the values of annuities to the children of those widows and widowers who lived to the end of each year; and, secondly, to ascertain the value of the annuities payable to those children whose parents both died in the same year.

The following is a specimen of the columns used in making the first of these calculations:—

AGE.		Annuity till Child reaches 21.		Previous Columns multiplied by Number of Widowers and Widows having Families.		Sum of two preceding Columns.	Previous Numbers Discounted.	Summation of the preceding Numbers.	Value of Annuity, 3 p. cent.
H.	W.	After Death of Widower.	After Death of Widow.						
27	20	1.489	1.187	33.791	29.822	63.613	62.680	2612.842	.46059
28	21	1.542	1.217	33.884	33.355	67.239	64.323	2550.162	.46997
29	22	1.586	1.253	34.507	38.635	73.142	67.932	2485.839	.47933
30	23	1.616	1.292	34.804	39.838	74.642	67.306	2417.907	.48840
31	24	1.646	1.337	35.090	41.225	76.315	66.810	2350.601	.49743

The first column after the age of the parents consists of the temporary annuities after the death of the widower until the youngest child reaches 21. The next column consists of the values of similar annuities after the death of the widows. The next two columns contain the products of these annuities, and the number of husbands and wives becoming widowers and widows in each year and living to the end of it, multiplied by the percentage having families under age. These numbers are summed together in the seventh column; and as the death will on an average occur in the middle of the year, the results now obtained are discounted for six months, and then multiplied by the value of £1, due 0, 1, 2, 3, &c. years hence. The ninth column contains the summation of the numbers in the eighth column, and these summations being divided by the number of pairs attaining each year of age discounted for 0, 1, 2, 3, &c. years, the result is the value of an annuity after the death of both parents who reach the first birthday after widowhood.

I now give a few lines of the second part of the calculation, to explain the process then adopted:—

WIFE'S AGE.	Annuity certain from Child's Age to 21.	Annuity certain, Parents Six Months' Older.	Previous Column Discounted Six Months.	Product of the previous Column and Number who both die Discounted, &c.	Summation of the Numbers in last Column.	Value of Annuity, 3 per cent.
20	15·415	15·415	15·189	2·602	141·240	·02490
21	15·415	15·415	15·189	2·526	138·638	·02555
22	15·415	15·415	15·189	3·270	136·112	·02625
23	15·415	15·415	15·189	2·381	132·842	·02683
24	15·415	15·415	15·189	2·312	130·461	·02761
25	15·415	15·146	14·924	5·304	128·149	·02843
26	14·877	14·877	14·659	4·046	122·845	·02857
* *	* * *	* * *	* * *	* * *	* * *	* * *

The first column shows the age of the mother, and the second the values of an annuity certain from youngest child's age until 21; for if both parents die in one year the charge on the Fund is immediate. As the deaths will probably be on an average deferred for six months, the third column contains the annuities

certain when the parents are half a year older. These values are discounted for six months in the next column; and, in the fifth column, they are multiplied by the number of those parents who both die in the same year and leave families, being also discounted for 0, 1, 2, 3, &c. years. The values now obtained are next summed together in the sixth column; and being divided by the product of the number of pairs commencing each year and the value of £1 due 0, 1, 2, 3, &c. years hence, the values are obtained of the annuities payable to the children of those parents who both die in the same year.

The results of these two calculations were added together, and form the twentieth Table.

TABLE XXI.—*Value of £1 payable at the Death of a Husband, if he die last, and leave a Family.*

The numbers who become widowers in each year having been ascertained, as well as the number of marriages dissolved in each year by the death of both parents, and it being assumed in regard to the latter that the instances in which the husband dies last is half the total number (which, although not strictly true, where there is a difference between the age of husband and wife, is quite correct enough for a Table of this kind), it is evident that the chance of the payment of £1 being made in the first year is expressed by

$$\frac{1}{\frac{1}{2}\delta_x \cdot \frac{1}{p_x} + r}$$

in which δ_x represents the number of marriages

dissolved by the death of both husband and wife, and p_x the number of pairs attaining the age of x .

The probability of the payment being made in the second year, if both lives drop, will be

$$\frac{\frac{1}{2}\delta_{x+1} \cdot \frac{1}{p_{x+1}}}{(1+r)^2},$$

and so on, the sum of the series being

$$\frac{\frac{1}{2}(\delta_x \cdot v^1 + \delta_{x+1} v^2 + \delta_{x+2} v^3 + \delta_{x+3} v^4 + \&c.)}{p_x}$$

Now, the number who became widowers in each year, and live to the end of it, being represented, as before, by $w \cdot w_{x+1} \cdot w_{x+2} \cdot \&c.$; and, it being assumed that at his wife's death each man will, on an average, be six months older than at the beginning of the year, the value of £1, payable at the death of any of those widowers whose wives die in the first year after x , will be

$$\frac{a_{x+\frac{1}{2}} \cdot w_x}{p_x} \times \frac{1}{(1+r)^{\frac{1}{2}}},$$

a_x representing the value of an assurance of £1, payable at the death of a person aged x . The value at the same date of £1, payable at the death of any of those who become widowers in the second year, will be

$$\frac{a_{x+1+\frac{1}{2}} \cdot w_{x+1}}{p_x} \cdot \frac{1}{(1+r)^{\frac{3}{2}}}, \&c., \&c.$$

Now, if the elements of which the above values are composed be arranged in parallel columns in the following manner, the mode of construction of Table XV. will be apparent.

HUSBAND'S AGE.	WIFE'S AGE.	Value of £1 at death of a person six months older than husband, discounted.	Widowers in each year having families.	Product of previous columns and value of £1 due 0, 1, 2, 3, &c. years hence.	Product of half number of both dead with families, and value of £1, due 1, 2, 3, &c. years hence.	Sum of the two last columns.	Additions of previous column.	Value of £1 at father's death, 3 per cent.
27	20	·37984	226·69	76·329	·831	77·160	4625·150	·08154
28	21	·38631	219·83	73·061	·807	73·868	4547·990	·08381
29	22	·39241	217·55	71·392	1·045	72·437	4474·122	·08627
30	23	·39834	215·27	69·769	·761	70·530	4401·685	·08891
31	24	·40439	212·98	68·193	·739	68·932	4331·155	·09167
32	25	·41068	311·47	98·530	1·725	100·255	4262·223	·09455
33	26	·41726	308·18	96·328	1·340	97·668	4161·968	·09678
•	•	• • •	• • •	• • •	• • •	• • •	• • •	• • •

The first two columns give the ages of husband and wife, the difference between them being seven years. The third column

contains the value of £1 at the death of a person six months older than the husband, discounted for half a year. In column fourth are given the number who become widowers in each year and live to the end of, and who have families; and these two columns being multiplied together, and discounted for 0, 1, 2, 3, 4, &c. years, make up the values in the fifth column. The next column contains the product of half the number of pairs dissolved in each year by the death of both husband and wife, and the values of £1 due in 1, 2, 3, 4, &c. years. The values at each age in the two columns last framed being added together, constitute the seventh column; and the eighth contains the summation of the previous column at each age and upwards, which being divided by the product of the pairs attaining each age, and the value of £1 due in 0, 1, 2, 3, 4, &c. years, produces the value of £1 payable at the death of the husband, if he be the last to die, and if he leave a family.

TABLES.

TABLE I.

55

First Marriages.—Schoolmasters' Widows' Fund.

AGE.	Number attaining each Age Un- married.	Number who Marry in the next Year.	Average Age of Wife taken by those who Marry at each Age.	Probability of Marriage.	Average Age at Marriage.	AGE.
20	2000	32.5	22	.7720000	31.456	20
21	1953.6	37.2	22	.7737000	31.692	21
22	1903.1	51.4	22	.7746835	31.949	22
23	1838.8	78.6	23	.7738200	32.290	23
24	1747.8	85.3	23	.7691384	32.804	24
25	1650.7	87	23	.7627064	33.367	25
26	1552.3	89.6	24	.7550085	33.951	26
27	1451.9	93.2	24	.7455060	34.567	27
28	1348.1	98.1	25	.7337733	35.233	28
29	1239.2	103	25	.7190930	35.975	29
30	1124.9	95.3	25	.7005957	36.821	30
31	1019.2	77.7	25	.6797490	37.690	31
32	931.9	71.7	25	.6600493	38.472	32
33	851.5	68.9	26	.6381674	39.260	33
34	774.7	62.5	26	.6124950	40.097	34
35	705	58.3	26	.5843971	40.946	35
36	640.1	53.5	26	.5525700	41.843	36
37	580.4	48.3	27	.5172295	42.795	37
38	526.3	40.6	27	.4786245	43.811	38
39	480.3	35.4	28	.4399334	44.831	39
40	439.6	27.9	28	.4001365	45.904	40
41	406.3	24.2	29	.3642629	46.923	41
42	376.8	22.7	29	.3285562	47.983	42
43	349	19.1	29	.2896849	49.214	43
44	325.1	13	29	.2522301	50.315	44
45	307.5	8.7	30	.2243903	51.684	45
46	294.4	7.3	30	.2048233	52.577	46
47	282.8	6.4	30	.1874116	53.413	47
48	272.4	5.3	30	.1710719	54.225	48
49	263.4	4.7	30	.1567957	54.960	49
50	255.2	4.4	31	.1434169	55.661	50
51	247.4	4.1	31	.1301536	56.366	51
52	239.8	3.8	32	.1171810	57.076	52
53	232.4	3.3	32	.1045611	57.792	53
54	225.4	2.9	33	.0931677	58.466	54
55	218.7	2.7	33	.0827618	59.102	55
56	212.1	2.6	34	.0726073	59.734	56
57	205.5	2.4	34	.0622871	60.390	57
58	198.9	2.2	35	.0522876	61.058	58
59	191.9	2.1	35	.0427306	61.744	59
60	184.4	1.7	36	.0330803	62.516	60
61	176.6	1.2	36	.0249151	63.295	61
62	169.1	1	36	.0189237	63.969	62
63	162.4	.8	36	.0135468	64.636	63
64	155.5	.6	36	.0090032	65.286	64
65	148.2	.5	36	.0053981	65.875	65
66	141.6	.3	36	.0021186	66.500	66
		1544.0				

TABLE II.

First Marriages.—Ministers' Widows' Fund.

AGE.	Number attaining each Age Un- married.	Number who Marry in the next Year.	Average Age of Wife taken by those who Marry at each Age.	Probability of Marriage.	Average Age at Marriage.	AGE.
20	2000	2.5	20	.7479500	34.866	20
21	1983.4	4.8	21	.7529494	34.889	21
22	1964.9	8.	21	.7575955	34.933	22
23	1943.2	22.	22	.7619389	35.000	23
24	1907.7	29.9	23	.7645855	35.173	24
25	1864.5	45.4	24	.7662642	35.397	25
26	1805.8	69.5	24	.7660317	35.721	26
27	1723.5	84.3	24	.7622860	36.209	27
28	1626.5	89.8	24	.7559174	36.806	28
29	1523.5	96.4	24	.7480800	37.461	29
30	1412.9	93.6	24	.7384104	38.197	30
31	1306.	91.5	25	.7271822	38.955	31
32	1202.1	86.2	25	.7139172	39.750	32
33	1104.5	80.5	25	.6989588	40.559	33
34	1013.7	74.8	26	.6821545	41.381	34
35	929.4	71.9	26	.6635433	42.216	35
36	848.7	66.2	27	.6419230	43.102	36
37	774.7	58.5	27	.6177874	44.012	37
38	708.4	55.2	27	.5930266	44.923	38
39	646.	51.8	28	.5648608	45.894	39
40	587.4	46.8	28	.5330270	46.626	40
41	533.4	36.3	28	.4992501	48.086	41
42	490.2	28.6	28	.4691963	49.126	42
43	455.	22.2	29	.4426374	50.067	43
44	426.5	19.6	29	.4201640	50.880	44
45	400.8	17.6	30	.3982036	51.664	45
46	377.5	15.2	30	.3761588	52.427	46
47	356.9	14.	30	.3552817	53.138	47
48	337.8	13.5	30	.3339254	53.838	48
49	319.8	13.3	30	.3105065	54.563	49
50	302.3	12.6	30	.2844857	55.575	50
51	285.9	11.4	31	.2567331	56.451	51
52	270.6	9.3	32	.2291205	57.361	52
53	257.9	8.	33	.2043428	58.219	53
54	245.9	6.5	34	.1817812	59.064	54
55	235.3	5.6	35	.1620017	59.840	55
56	225.6	5.	35	.1445036	60.586	56
57	216.4	4.7	35	.1275416	61.326	57
58	207.3	3.9	35	.1104676	62.112	58
59	198.5	3.2	35	.0957179	62.852	59
60	189.8	2.9	36	.0832455	63.531	60
61	180.6	2.7	36	.0714286	64.213	61
62	171.5	2.3	36	.0594752	64.931	62
63	162.9	2.	36	.0484960	65.639	63
64	154.7	1.6	36	.0381383	66.365	64
65	147.	1.3	36	.0292517	67.058	65
66	139.7	1.	36	.0214746	67.733	66
67	132.8	.8	36	.0150602	68.350	67
68	126.1	.7	36	.0095163	68.917	68
69	119.6	.5	36	.0041806	69.500	69
		1495.9				

TABLE III.

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Probability of the Re-marriage of Widowers.

AGE.	Value of Probability of Re-marriage.	Average Age of Wife taken by those who Marry at each Age.
28	.916747	27
29	.915376	27
30	.910808	27
31	.906759	27
32	.901536	27
33	.893569	27
34	.880650	27
35	.863255	28
36	.847118	28
37	.834738	28
38	.822638	28
39	.805848	28
40	.789261	31
41	.771363	31
42	.753617	31
43	.734694	31
44	.712259	31
45	.685510	31
46	.656805	31
47	.621071	31
48	.583977	31
49	.547129	31
50	.517242	36
51	.485714	36
52	.452482	36
53	.418856	36
54	.388333	36
55	.355596	36
56	.317647	36
57	.283298	36
58	.255079	36
59	.223301	36
60	.189474	41
61	.158640	41
62	.128441	41
63	.096027	41
64	.073944	41
65	.056180	41
66	.040001	41
67	.029536	41
68	.017857	41
69	.009434	41
70	.004975	41

Value of £1, payable on the Marriage of Males at present *Unmarried*,
according to the experience of the *Schoolmasters' Widows' Fund*.

AGE.	VALUE OF £1 AT MARRIAGE.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
20	·56193	·53539	·51072	·46627	20
21	·57565	·55037	·52679	·48416	21
22	·58883	·56487	·54248	·50185	22
23	·59932	·57664	·55540	·51671	23
24	·60377	·58214	·56182	·52472	24
25	·60604	·58541	·56600	·53045	25
26	·60692	·58730	·56880	·53486	26
27	·60573	·58712	·56954	·53722	27
28	·60179	·58413	·56744	·53669	28
29	·59397	·57718	·56128	·53196	29
30	·58104	·56494	·54961	·52151	30
31	·56566	·55025	·53563	·50861	31
32	·55257	·53801	·52420	·49862	32
33	·53748	·52381	·51083	·48677	33
34	·51819	·50538	·49319	·47062	34
35	·49659	·48463	·47329	·45224	35
36	·47093	·45982	·44927	·42971	36
37	·44148	·43117	·42139	·40325	37
38	·40841	·39885	·38979	·37299	38
39	·37511	·36630	·35796	·34250	39
40	·34049	·33239	·32471	·31050	40
41	·30971	·30232	·29532	·28236	41
42	·27894	·27220	·26582	·25403	42
43	·24427	·23809	·23224	·22143	43
44	·21039	·20469	·19929	·18932	44
45	·18636	·18114	·17618	·16702	45
46	·17054	·16580	·16129	·15293	46
47	·15665	·15236	·14828	·14070	47
48	·14374	·13989	·13622	·12939	48
49	·13252	·12909	·12582	·11971	49
50	·12215	·11913	·11624	·11083	50
51	·11181	·10917	·10664	·10189	51
52	·10145	·09917	·09698	·09285	52
53	·09142	·08947	·08759	·08404	53
54	·08205	·08040	·07881	·07580	54
55	·07373	·07237	·07105	·06854	55
56	·06527	·06416	·06303	·06105	56
57	·05666	·05578	·05493	·05331	57
58	·04827	·04760	·04694	·04569	58
59	·03970	·03920	·03871	·03778	59
60	·03075	·03039	·03004	·02937	60
61	·02330	·02305	·02281	·02234	61
62	·01787	·01770	·01753	·01723	62
63	·01291	·01281	·01271	·01253	63
64	·00867	·00862	·00856	·00846	64
65	·00526	·00524	·00521	·00517	65
66	·00209	·00208	·00207	·00206	66

TABLE V.

Value of £1, payable on the Marriage of Males at present *Unmarried*, according to the experience of the *Ministers' Widows' Fund*.

AGE.	VALUE OF £1 AT MARRIAGE.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
20	·49388	·46340	·43539	·38586	20
21	·51170	·48237	·45534	·40727	21
22	·52954	·50147	·47552	·42916	22
23	·54735	·52065	·49589	·45146	23
24	·56254	·53715	·51354	·47101	24
25	·57655	·55250	·53008	·48958	25
26	·58766	·56488	·54360	·50504	26
27	·59325	·57152	·55119	·51427	27
28	·59489	·57408	·55458	·51910	28
29	·59435	·57439	·55566	·52152	29
30	·59087	·57163	·55356	·52057	30
31	·58568	·56716	·54975	·51792	31
32	·57815	·56032	·54355	·51285	32
33	·56891	·55179	·53567	·50613	33
34	·55788	·54148	·52603	·49769	34
35	·54507	·52940	·51463	·48753	35
36	·52883	·51385	·49973	·47380	36
37	·51001	·49571	·48223	·45747	37
38	·49060	·47709	·46426	·44070	38
39	·46751	·45456	·44234	·41991	39
40	·44012	·42774	·41606	·39459	40
41	·41018	·39829	·38705	·36638	41
42	·38459	·37324	·36251	·34277	42
43	·36293	·35219	·34203	·32330	43
44	·34598	·33594	·32641	·30883	44
45	·32958	·32025	·31138	·29497	45
46	·31311	·30449	·29628	·28107	46
47	·29800	·29011	·28259	·26863	47
48	·28235	·27520	·26837	·25567	48
49	·26435	·25793	·25177	·24032	49
50	·24340	·23765	·23215	·22188	50
51	·22036	·21525	·21035	·20119	51
52	·19706	·19253	·18817	·18004	52
53	·17637	·17240	·16857	·16141	53
54	·15751	·15405	·15070	·14442	54
55	·14121	·13823	·13534	·12990	55
56	·12684	·12429	·12180	·11713	56
57	·11258	·11044	·10834	·10438	57
58	·09790	·09611	·09435	·09104	58
59	·08532	·08385	·08239	·07953	59
60	·07495	·07378	·07259	·07036	60
61	·06462	·06369	·06274	·06097	61
62	·05420	·05349	·05275	·05137	62
63	·04465	·04413	·04357	·04254	63
64	·03519	·03479	·03441	·03367	64
65	·02725	·02698	·02671	·02620	65
66	·02040	·02023	·02006	·01973	66
67	·01480	·01470	·01461	·01443	67
68	·00937	·00933	·00928	·00921	68
69	·00435	·00434	·00433	·00431	69

Value of an Annuity to the Widows who may be left by Males at present *Unmarried*, according to the experience of the *School-masters' Widows' Fund*.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
20	2·4895	2·0562	1·7015	1·1935	20
21	2·5649	2·1264	1·7663	1·2481	21
22	2·6362	2·1962	1·8310	1·3032	22
23	2·7064	2·2598	1·8902	1·3543	23
24	2·7600	2·3107	1·9377	1·3952	24
25	2·8056	2·3542	1·9792	1·4315	25
26	2·8442	2·3916	2·0158	1·4643	26
27	2·8790	2·4254	2·0491	1·4941	27
28	2·9010	2·4481	2·0728	1·5162	28
29	2·9145	2·4628	2·0887	1·5315	29
30	2·9075	2·4597	2·0881	1·5338	30
31	2·8865	2·4447	2·0777	1·5290	31
32	2·8685	2·4325	2·0711	1·5280	32
33	2·8362	2·4083	2·0545	1·5194	33
34	2·7902	2·3719	2·0269	1·5018	34
35	2·7266	2·3208	1·9863	1·4751	35
36	2·6358	2·2465	1·9252	1·4329	36
37	2·5170	2·1473	1·8429	1·3739	37
38	2·3820	2·0334	1·7470	1·3035	38
39	2·2342	1·9085	1·6416	1·2255	39
40	2·0807	1·7784	1·5304	1·1431	40
41	1·9371	1·6577	1·4272	1·0678	41
42	1·7948	1·5376	1·3240	·9922	42
43	1·6240	1·3920	1·1983	·8986	43
44	1·4492	1·2422	1·0692	·8013	44
45	1·3227	1·1351	·9776	·7336	45
46	1·2433	1·0690	·9221	·6940	46
47	1·1708	1·0090	·8718	·6589	47
48	1·0999	·9502	·8227	·6247	48
49	1·0352	·8967	·7783	·5941	49
50	·9717	·8442	·7348	·5638	50
51	·9071	·7902	·6897	·5320	51
52	·8380	·7322	·6407	·4970	52
53	·7699	·6746	·5919	·4616	53
54	·7026	·6174	·5432	·4260	54
55	·6422	·5659	·4994	·3939	55
56	·5769	·5098	·4513	·3579	56
57	·5092	·4511	·4005	·3193	57
58	·4397	·3906	·3477	·2787	58
59	·3678	·3274	·2922	·2352	59
60	·2893	·2581	·2308	·1865	60
61	·2239	·2002	·1793	·1453	61
62	·1749	·1566	·1406	·1145	62
63	·1287	·1155	·1039	·0850	63
64	·0880	·0791	·0714	·0586	64
65	·0543	·0489	·0442	·0365	65
66	·0219	·0198	·0179	·0149	66

TABLE VII.

Value of an Annuity to the Widows who may be left by Males at present *Unmarried*, according to the experience of the *Ministers' Widows' Fund*.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
20	2·3635	1·9238	1·5704	1·0706	20
21	2·4501	2·0037	1·6433	1·1309	21
22	2·5380	2·0853	1·7182	1·1933	22
23	2·6273	2·1685	1·7948	1·2577	23
24	2·7109	2·2465	1·8670	1·3187	24
25	2·7934	2·3237	1·9387	1·3796	25
26	2·8709	2·3961	2·0059	1·4367	26
27	2·9327	2·4538	2·0601	1·4827	27
28	2·9814	2·4996	2·1036	1·5197	28
29	3·0213	2·5381	2·1402	1·5515	29
30	3·0496	2·5662	2·1675	1·5761	30
31	3·0686	2·5860	2·1883	1·5956	31
32	3·0842	2·6023	2·2060	1·6123	32
33	3·0891	2·6095	2·2162	1·6233	33
34	3·0809	2·6062	2·2174	1·6280	34
35	3·0692	2·5998	2·2154	1·6304	35
36	3·0365	2·5754	2·1976	1·6207	36
37	2·9957	2·5438	2·1730	1·6057	37
38	2·9448	2·5033	2·1412	1·5850	38
39	2·8673	2·4394	2·0892	1·5480	39
40	2·7729	2·3607	2·0226	1·4996	40
41	2·6579	2·2644	1·9399	1·4393	41
42	2·5556	2·1800	1·8684	1·3886	42
43	2·4660	2·1067	1·8077	1·3466	43
44	2·4032	2·0569	1·7680	1·3212	44
45	2·3375	2·0051	1·7267	1·2952	45
46	2·2723	1·9536	1·6857	1·2694	46
47	2·2086	1·9038	1·6463	1·2457	47
48	2·1351	1·8455	1·5997	1·2169	48
49	2·0391	1·7671	1·5357	1·1742	49
50	1·9148	1·6637	1·4495	1·1137	50
51	1·7659	1·5383	1·3434	1·0373	51
52	1·6116	1·4074	1·2317	·9554	52
53	1·4724	1·2596	1·1306	·8809	53
54	1·3443	1·1795	1·0368	·8115	54
55	1·2324	1·0838	·9548	·7503	55
56	1·1336	·9992	·8822	·6963	56
57	1·0290	·9089	·8043	·6374	57
58	·9145	·8092	·7176	·5708	58
59	·8119	·7199	·6397	·5108	59
60	·7240	·6434	·5731	·4598	60
61	·6362	·5666	·5038	·4073	61
62	·5443	·4858	·4345	·3513	62
63	·4573	·4090	·3666	·2976	63
64	·3680	·3297	·2961	·2413	64
65	·2907	·2610	·2344	·1922	65
66	·2218	·1997	·1801	·1480	66
67	·1637	·1477	·1336	·1104	67
68	·1054	·0954	·0865	·0719	68
69	·0497	·0451	·0410	·0343	69

Value of an Annuity to the Widows who may be left from Re-marriage of Widowers, according to the experience of the *Ministers' and Schoolmasters' Widows' Funds*.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
28	3·4782	2·9542	2·5232	1·8684	28
29	3·5653	3·0324	2·5948	1·9264	29
30	3·6357	3·0941	2·6490	1·9681	30
31	3·7103	3·1615	2·7089	2·0169	31
32	3·7795	3·2243	2·7653	2·0632	32
33	3·8328	3·2720	2·8089	2·0977	33
34	3·8540	3·2897	2·8258	2·1085	34
35	3·8405	3·2771	2·8144	2·0969	35
36	3·8518	3·2889	2·8227	2·1043	36
37	3·8788	3·3175	2·8488	2·1293	37
38	3·9021	3·3432	2·8742	2·1544	38
39	3·8879	3·3355	2·8700	2·1548	39
40	3·8656	3·3226	2·8616	2·1540	40
41	3·8813	3·3387	2·8781	2·1689	41
42	3·8958	3·3560	2·8957	2·1865	42
43	3·9004	3·3651	2·9073	2·2014	43
44	3·8800	3·3518	2·9004	2·2022	44
45	3·8294	3·3120	2·8709	2·1851	45
46	3·7585	3·2561	2·8274	2·1585	46
47	3·6307	3·1511	2·7399	2·0977	47
48	3·4814	3·0276	2·6372	2·0268	48
49	3·3136	2·8881	2·5215	1·9463	49
50	3·1786	2·7781	2·4329	1·8884	50
51	3·0850	2·7021	2·3712	1·8480	51
52	2·9711	2·6079	2·2934	1·7947	52
53	2·8375	2·4957	2·2000	1·7287	53
54	2·7053	2·3851	2·1082	1·6643	54
55	2·5509	2·2545	1·9980	1·5846	55
56	2·3406	2·0734	1·8418	1·4673	56
57	2·1354	1·8964	1·6890	1·3522	57
58	1·9519	1·7385	1·5532	1·2504	58
59	1·7197	1·5359	1·3766	1·1140	59
60	1·4537	1·3019	1·1708	·9525	60
61	1·2566	1·1274	1·0158	·8291	61
62	1·0477	·9414	·8497	·6956	62
63	·8069	·7259	·6558	·5379	63
64	·6424	·5791	·5241	·4314	64
65	·5031	·4544	·4121	·3406	65
66	·3679	·3328	·3024	·2508	66
67	·2799	·2539	·2314	·1929	67
68	·1746	·1585	·1449	·1213	68
69	·0951	·0866	·0794	·0665	69
70	·0517	·0473		·0367	70

TABLE IX.

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Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

EQUAL AGES.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
21	·64614	·51493	·40996	·26650	21
22	·65084	·51945	·41470	·27068	22
23	·65535	·52410	·41961	·27508	23
24	·65995	·52889	·42471	·27969	24
25	·66465	·53382	·43001	·28454	25
26	·66909	·53857	·43523	·28942	26
27	·67362	·54346	·44065	·29455	27
28	·67749	·54819	·44573	·29951	28
29	·67883	·55077	·44896	·30313	29
30	·67706	·55071	·44994	·30510	30
31	·67382	·54941	·44989	·30634	31
32	·66945	·54718	·44907	·30605	32
33	·66449	·54449	·44790	·30557	33
34	·65926	·54166	·44663	·30513	34
35	·65365	·53856	·44521	·30470	35
36	·64737	·53494	·44341	·30408	36
37	·63971	·53014	·44064	·30278	37
38	·63062	·52412	·43684	·30073	38
39	·62039	·51712	·43222	·29812	39
40	·60799	·50822	·42599	·29532	40
41	·59170	·49597	·41686	·29235	41
42	·57218	·48100	·40528	·28954	42
43	·54979	·46328	·39148	·28301	43
44	·52575	·44419	·37636	·27344	44
45	·50039	·42391	·36014	·26297	45
46	·47452	·40309	·34339	·25204	46
47	·44896	·38245	·32674	·24111	47
48	·42465	·36278	·31087	·23069	48
49	·40299	·34534	·29687	·22161	49
50	·38258	·32887	·28361	·21199	50
51	·36290	·31296	·27079	·20463	51
52	·34162	·29556	·25659	·19511	52
53	·31902	·27690	·24120	·18457	53
54	·29506	·25693	·22454	·17289	54
55	·27061	·23639	·20729	·16059	55
56	·24628	·21584	·18990	·14806	56
57	·22224	·19540	·17252	·13536	57
58	·19712	·17386	·15401	·12157	58
59	·17058	·15088	·13406	·10642	59
60	·14377	·12751	·11359	·09063	60
61	·11498	·10221	·09127	·07314	61
62	·08850	·07884	·07056	·05678	62
63	·06702	·05986	·05371	·04344	63
64	·04908	·04395	·03954	·03216	64
65	·03400	·03052	·02754	·02252	65
66	·02232	·02010	·01820	·01497	66
67	·01265	·01142	·01037	·00857	67
68	·00616	·00558	·00508	·00422	68
69	·00234	·00213	·00195	·00163	69

Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

DIFFERENCE OF AGE, ONE YEAR.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
21	·63213	·50372	·40128	·26097	21
22	·63622	·50786	·40560	·26480	22
23	·64031	·51209	·41002	·26888	23
24	·64448	·51640	·41477	·27316	24
25	·64857	·52082	·41958	·27763	25
26	·65243	·52509	·42434	·28214	26
27	·65635	·52948	·42926	·28687	27
28	·65972	·53375	·43389	·29149	28
29	·66097	·53621	·43703	·29498	29
30	·65955	·53642	·43822	·29708	30
31	·65681	·53551	·43848	·29853	31
32	·65302	·53374	·43803	·29863	32
33	·64855	·53144	·43717	·29850	33
34	·64378	·52898	·43620	·29840	34
35	·63851	·52614	·43498	·29823	35
36	·63220	·52245	·43310	·29766	36
37	·62410	·51724	·42995	·29618	37
38	·61456	·51079	·42575	·29394	38
39	·60994	·50342	·42077	·29116	39
40	·59149	·49443	·41443	·28823	40
41	·57571	·48258	·40560	·28516	41
42	·55704	·46829	·39459	·28216	42
43	·53580	·45153	·38158	·27590	43
44	·51292	·43341	·36727	·26690	44
45	·48872	·41410	·35187	·25702	45
46	·46391	·39417	·33586	·24663	46
47	·43909	·37415	·31973	·23607	47
48	·41491	·35456	·30391	·22566	48
49	·39274	·33665	·28948	·21622	49
50	·37144	·31938	·27549	·20700	50
51	·35064	·30245	·26175	·19789	51
52	·32842	·28419	·24676	·18770	52
53	·30515	·26490	·23077	·17664	53
54	·28079	·24453	·21372	·16459	54
55	·25620	·22381	·19627	·15207	55
56	·23216	·20347	·17902	·13958	56
57	·20868	·18348	·16199	·12709	57
58	·18456	·16278	·14419	·11381	58
59	·15936	·14095	·12523	·09939	59
60	·13418	·11899	·10599	·08455	60
61	·10746	·09552	·08529	·06834	61
62	·08297	·07391	·06614	·05321	62
63	·06309	·05635	·05055	·04088	63
64	·04645	·04160	·03742	·03043	64
65	·03241	·02909	·02625	·02146	65
66	·02143	·01929	·01747	·01437	66
67	·01222	·01103	·01002	·00828	67
68	·00595	·00539	·00491	·00408	68
69	·00226	·00205	·00188	·00157	69

TABLE XI.

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Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

DIFFERENCE OF AGE, TWO YEARS.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
21	·61782	·49251	·39260	·25544	21
22	·62160	·49627	·39649	·25892	22
23	·62526	·50008	·40043	·26268	23
24	·62901	·50391	·40484	·26663	24
25	·63249	·50782	·40916	·27072	25
26	·63577	·51161	·41345	·27486	26
27	·63908	·51550	·41788	·27920	27
28	·64195	·51931	·42211	·28347	28
29	·64311	·52165	·42510	·28684	29
30	·64204	·52213	·42650	·28906	30
31	·63980	·52161	·42707	·29072	31
32	·63659	·52030	·42699	·29121	32
33	·63261	·51839	·42645	·29143	33
34	·62831	·51630	·42577	·29167	34
35	·62338	·51372	·42475	·29176	35
36	·61703	·50996	·42279	·29124	36
37	·60849	·50434	·41926	·28958	37
38	·59850	·49746	·41466	·28715	38
39	·58749	·48972	·40932	·28420	39
40	·57499	·48064	·40287	·28114	40
41	·55972	·46919	·39435	·27797	41
42	·54190	·45558	·38390	·27478	42
43	·52181	·43978	·37168	·26879	43
44	·50009	·42263	·35818	·26036	44
45	·47706	·40429	·34360	·25107	45
46	·45331	·38525	·32834	·24122	46
47	·42922	·36585	·31272	·23103	47
48	·40517	·34634	·29695	·22063	48
49	·38249	·32797	·28209	·21083	49
50	·36030	·30989	·26737	·20101	50
51	·33838	·29194	·25271	·19115	51
52	·31523	·27282	·23693	·18029	52
53	·29129	·25290	·22034	·16871	53
54	·26652	·23213	·20290	·15629	54
55	·24179	·21123	·18525	·14355	55
56	·21804	·19110	·16814	·13110	56
57	·19512	·17156	·15146	·11882	57
58	·17201	·15170	·13437	·10605	58
59	·14814	·13102	·11640	·09237	59
60	·12459	·11047	·09840	·07848	60
61	·09995	·08883	·07931	·06354	61
62	·07744	·06898	·06172	·04964	62
63	·05916	·05284	·04739	·03832	63
64	·04382	·03925	·03530	·02870	64
65	·03082	·02767	·02496	·02041	65
66	·02054	·01849	·01674	·01377	66
67	·01179	·01064	·00967	·00799	67
68	·00574	·00520	·00474	·00394	68
69	·00218	·00197	·00181	·00151	69

Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

DIFFERENCE OF AGE, THREE YEARS.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent	
21	·60351	·48130	·38392	·24991	21
22	·60698	·48468	·38738	·25304	22
23	·61021	·48807	·39084	·25648	23
24	·61355	·49142	·39491	·26010	24
25	·61641	·49482	·39874	·26381	25
26	·61911	·49813	·40256	·26758	26
27	·62181	·50152	·40650	·27153	27
28	·62418	·50487	·41033	·27545	28
29	·62525	·50709	·41317	·27870	29
30	·62454	·50784	·41478	·28104	30
31	·62279	·50772	·41566	·28291	31
32	·62016	·50687	·41596	·28379	32
33	·61667	·50535	·41573	·28436	33
34	·61284	·50362	·41534	·28494	34
35	·60825	·50130	·41452	·28529	35
36	·60186	·49748	·41248	·28482	36
37	·59289	·49145	·40857	·28299	37
38	·58244	·48414	·40357	·28036	38
39	·57104	·47602	·39787	·27724	39
40	·55849	·46685	·39131	·27405	40
41	·54374	·45580	·38310	·27078	41
42	·52676	·44287	·37321	·26740	42
43	·50782	·42803	·36178	·26168	43
44	·48726	·41185	·34909	·25382	44
45	·46540	·39449	·33533	·24512	45
46	·44271	·37634	·32082	·23581	46
47	·41935	·35755	·30571	·22599	47
48	·39543	·33813	·28999	·21560	48
49	·37224	·31929	·27470	·20544	49
50	·34916	·30040	·25925	·19502	50
51	·32612	·28143	·24367	·18441	51
52	·30204	·26145	·22710	·17288	52
53	·27743	·24090	·20992	·16078	53
54	·25225	·21973	·19208	·14799	54
55	·22738	·19865	·17423	·13503	55
56	·20392	·17873	·15726	·12262	56
57	·18157	·15964	·14093	·11056	57
58	·15946	·14062	·12455	·09829	58
59	·13692	·12109	·10757	·08535	59
60	·11500	·10196	·09081	·07241	60
61	·09244	·08214	·07333	·05874	61
62	·07191	·06405	·05730	·04608	62
63	·05523	·04933	·04424	·03576	63
64	·04120	·03690	·03318	·02697	64
65	·02924	·02625	·02368	·01936	65
66	·01965	·01769	·01601	·01317	66
67	·01136	·01026	·00932	·00770	67
68	·00554	·00502	·00457	·00380	68
69	·00210	·00190		·00145	69

TABLE XIII.

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Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

DIFFERENCE OF AGE, FOUR YEARS.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
22	·59236	·47309	·37827	·24716	22
23	·59516	·47606	·38125	·25028	23
24	·59809	·47893	·38498	·25358	24
25	·60033	·48182	·38832	·25690	25
26	·60245	·48465	·39167	·26030	26
27	·60454	·48754	·39512	·26386	27
28	·60642	·49043	·39850	·26743	28
29	·60739	·49253	·40124	·27056	29
30	·60704	·49355	·40306	·27302	30
31	·60578	·49383	·40425	·27510	31
32	·60373	·49344	·40493	·27638	32
33	·60073	·49231	·40501	·27730	33
34	·59737	·49094	·40491	·27821	34
35	·59312	·48888	·40429	·27882	35
36	·58669	·48500	·40217	·27841	36
37	·57729	·47856	·39788	·27640	37
38	·56638	·47082	·39248	·27357	38
39	·55459	·46232	·38643	·27028	39
40	·54199	·45306	·37975	·26696	40
41	·52774	·44241	·37185	·26359	41
42	·51163	·43016	·36253	·26002	42
43	·49383	·41628	·35188	·25457	43
44	·47443	·40107	·34000	·24728	44
45	·45374	·38469	·32706	·23917	45
46	·43211	·36743	·31330	·23040	46
47	·40949	·34925	·29870	·22095	47
48	·38569	·32992	·28303	·21057	48
49	·36199	·31061	·26731	·20005	49
50	·33803	·29091	·25113	·18904	50
51	·31386	·27093	·23463	·17767	51
52	·28885	·25009	·21727	·16548	52
53	·26357	·22890	·19950	·15285	53
54	·23799	·20733	·18126	·13969	54
55	·21297	·18608	·16321	·12651	55
56	·18980	·16636	·14638	·11414	56
57	·16802	·14772	·13040	·10230	57
58	·14691	·12954	·11473	·09053	58
59	·12570	·11116	·09874	·07833	59
60	·10541	·09345	·08322	·06634	60
61	·08493	·07546	·06735	·05394	61
62	·06638	·05912	·05288	·04252	62
63	·05130	·04582	·04109	·03320	63
64	·03858	·03455	·03106	·02524	64
65	·02766	·02483	·02240	·01831	65
66	·01876	·01689	·01528	·01257	66
67	·01094	·00988	·00897	·00741	67
68	·00534	·00484	·00440	·00366	68
69	·00202	·00183	·00167	·00140	69

Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

DIFFERENCE OF AGE, FIVE YEARS.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
22	·57774	·46150	·36916	·24128	22
23	·58011	·46405	·37166	·24408	23
24	·58263	·46644	·37505	·24706	24
25	·58425	·46882	·37790	·24999	25
26	·58579	·47117	·38078	·25302	26
27	·58728	·47356	·38374	·25619	27
28	·58866	·47599	·38667	·25962	28
29	·58953	·47797	·38931	·26242	29
30	·58954	·47927	·39134	·26501	30
31	·58878	·47994	·39285	·26729	31
32	·58730	·48001	·39390	·26897	32
33	·58479	·47927	·39429	·27024	33
34	·58190	·47826	·39448	·27148	34
35	·57799	·47646	·39406	·27235	35
36	·57153	·47252	·39186	·27200	36
37	·56169	·46567	·38719	·26981	37
38	·55032	·45750	·38139	·26678	38
39	·53815	·44862	·37499	·26333	39
40	·52549	·43928	·36820	·25988	40
41	·51175	·42902	·36060	·25640	41
42	·49650	·41745	·35185	·25264	42
43	·47984	·40454	·34188	·24746	43
44	·46160	·39029	·33091	·24075	44
45	·44208	·37489	·31879	·23323	45
46	·42151	·35852	·30578	·22500	46
47	·39963	·34095	·29169	·21591	47
48	·37595	·32171	·27608	·20555	48
49	·35175	·30193	·25992	·19467	49
50	·32690	·28142	·24302	·18306	50
51	·30160	·26043	·22559	·17093	51
52	·27566	·23873	·20744	·15808	52
53	·24971	·21691	·18908	·14492	53
54	·22373	·19493	·17044	·13139	54
55	·19856	·17351	·15219	·11799	55
56	·17569	·15399	·13550	·10566	56
57	·15447	·13580	·11987	·09404	57
58	·13436	·11846	·10491	·08277	58
59	·11449	·10123	·08991	·07131	59
60	·09582	·08494	·07563	·06027	60
61	·07742	·06878	·06138	·04914	61
62	·06085	·05419	·04846	·03896	62
63	·04738	·04231	·03794	·03065	63
64	·03596	·03220	·02895	·02352	64
65	·02608	·02341	·02112	·01726	65
66	·01787	·01609	·01456	·01197	66
67	·01052	·00950	·00862	·00712	67
68	·00514	·00466	·00424	·00352	68
69	·00194	·00176	·00160	·00135	69

TABLE XV.

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Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

DIFFERENCE OF AGE, SIX YEARS.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
23	·56506	·45204	·36207	·23788	23
24	·56717	·45396	·36512	·24054	24
25	·56817	·45582	·36748	·24309	25
26	·56913	·45770	·36989	·24575	26
27	·57002	·45958	·37236	·24852	27
28	·57090	·46155	·37494	·25141	28
29	·57167	·46342	·37738	·25428	29
30	·57204	·46499	·37962	·25700	30
31	·57178	·46605	·38145	·25948	31
32	·57087	·46658	·38287	·26156	32
33	·56886	·46623	·38357	·26318	33
34	·56643	·46559	·38406	·26475	34
35	·56286	·46404	·38383	·26589	35
36	·55637	·46004	·38156	·26559	36
37	·54609	·45278	·37650	·26322	37
38	·53426	·44418	·37030	·25999	38
39	·52171	·43492	·36355	·25638	39
40	·50900	·42550	·35665	·25280	40
41	·49576	·41563	·34935	·24921	41
42	·48137	·40474	·34117	·24526	42
43	·46585	·39280	·33199	·24035	43
44	·44878	·37952	·32182	·23422	44
45	·43042	·36509	·31052	·22729	45
46	·41091	·34961	·29826	·21960	46
47	·38977	·33265	·28468	·21087	47
48	·36621	·31350	·26913	·20053	48
49	·34151	·29325	·25254	·18929	49
50	·31577	·27194	·23491	·17708	50
51	·28935	·24993	·21656	·16419	51
52	·26247	·22737	·19762	·15068	52
53	·23585	·20492	·17866	·13700	53
54	·20947	·18253	·15963	·12309	54
55	·18415	·16094	·14117	·10947	55
56	·16158	·14163	·12462	·09719	56
57	·14092	·12388	·10935	·08578	57
58	·12181	·10739	·09509	·07501	58
59	·10328	·09130	·08108	·06429	59
60	·08623	·07643	·06804	·05420	60
61	·06991	·06210	·05541	·04434	61
62	·05532	·04926	·04405	·03540	62
63	·04346	·03880	·03479	·02810	63
64	·03334	·02985	·02684	·02180	64
65	·02450	·02199	·01984	·01621	65
66	·01698	·01529	·01388	·01138	66
67	·01010	·00912	·00828	·00684	67
68	·00494	·00448	·00408	·00338	68
69	·00186	·00169	·00154	·00130	69

TABLE XVI.

Value of an Annuity to the Widows who may be left through the
Second Marriage of Males whose First Wife is still alive.

DIFFERENCE OF AGE, SEVEN YEARS.

AGE.	VALUE OF A SURVIVORSHIP ANNUITY OF £1.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
24	·55171	·44148	·35519	·23402	24
25	·55209	·44283	·35706	·23619	25
26	·55247	·44423	·35901	·23848	26
27	·55276	·44560	·36098	·24086	27
28	·55314	·44711	·36311	·24340	28
29	·55382	·44887	·36546	·24614	29
30	·55454	·45071	·36790	·24899	30
31	·55478	·45216	·37005	·25167	31
32	·55444	·45315	·37184	·25415	32
33	·55293	·45319	·37285	·25612	33
34	·55096	·45292	·37364	·25802	34
35	·54773	·45163	·37361	·25943	35
36	·54121	·44756	·37126	·25918	36
37	·53049	·43989	·36582	·25663	37
38	·51821	·43086	·35922	·25321	38
39	·50527	·42123	·35211	·24943	39
40	·49251	·41172	·34510	·24572	40
41	·47977	·40224	·33810	·24203	41
42	·46624	·39203	·33049	·23789	42
43	·45186	·38106	·32220	·23324	43
44	·43596	·36875	·31274	·22769	44
45	·41876	·35529	·30225	·22135	45
46	·40031	·34070	·29074	·21420	46
47	·37991	·32436	·27767	·20583	47
48	·35647	·30529	·26218	·19552	48
49	·33127	·28457	·24516	·18391	49
50	·30464	·26246	·22680	·17110	50
51	·27710	·23943	·20753	·15745	51
52	·24928	·21601	·18780	·14328	52
53	·22199	·19293	·16824	·12908	53
54	·19521	·17014	·14882	·11480	54
55	·16975	·14837	·13015	·10095	55
56	·14747	·12927	·11375	·08872	56
57	·12737	·11197	·09883	·07752	57
58	·10926	·09632	·08528	·06726	58
59	·09207	·08138	·07225	·05727	59
60	·07665	·06792	·06045	·04813	60
61	·06240	·05542	·04944	·03954	61
62	·04980	·04433	·03964	·03184	62
63	·03954	·03529	·03164	·02555	63
64	·03072	·02750	·02473	·02008	64
65	·02292	·02057	·01856	·01516	65
66	·01610	·01449	·01312	·01079	66
67	·00968	·00874	·00794	·00656	67
68	·00474	·00430	·00392	·00325	68
69	·00178	·00162	·00148	·00125	69

TABLE XVII.

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Value of an Annuity of £1 to the Widows who may be left through
the Second Marriage of Males now *Unmarried*.

AGE.	VALUE OF SURVIVORSHIP ANNUITY.				AGE.
	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	
20	·25859	·19934	·15432	·09406	20
21	·26788	·20748	·16139	·09929	21
22	·27715	·21567	·16854	·10465	22
23	·28635	·22387	·17575	·11012	23
24	·29397	·23082	·18198	·11500	24
25	·30081	·23721	·18782	·11968	25
26	·30583	·24216	·19251	·12364	26
27	·30742	·24434	·19496	·12611	27
28	·30644	·24445	·19572	·12746	28
29	·30387	·24325	·19541	·12811	29
30	·29911	·24025	·19363	·12774	30
31	·29296	·23611	·19089	·12672	31
32	·28499	·23044	·18690	·12484	32
33	·27563	·22361	·18192	·12226	33
34	·26485	·21557	·17593	·11897	34
35	·25267	·20632	·16890	·11491	35
36	·23819	·19511	·16019	·10963	36
37	·22231	·18266	·15040	·10353	37
38	·20647	·17017	·14053	·09731	38
39	·18883	·15611	·12928	·09005	39
40	·16928	·14035	·11653	·08163	40
41	·14872	·12364	·10291	·07246	41
42	·13115	·10934	·09122	·06459	42
43	·11622	·09719	·08127	·05789	43
44	·10426	·08746	·07332	·05255	44
45	·09307	·07832	·06582	·04748	45
46	·08240	·06956	·05858	·04253	46
47	·07295	·06177	·05213	·03809	47
48	·06394	·05436	·04590	·03376	48
49	·05484	·04677	·03951	·02924	49
50	·04556	·03897	·03287	·02449	50
51	·03660	·03140	·02639	·01978	51
52	·02859	·02458	·02051	·01544	52
53	·02228	·01919	·01583	·01198	53
54	·01718	·01481	·01209	·00924	54
55	·01334	·01152	·00940	·00717	55
56	·01037	·00896	·00730	·00564	56
57	·00789	·00682	·00545	·00417	57
58	·00574	·00495	·00373	·00299	58
59	·00416	·00359	·00281	·00192	59

TABLE XVIII.

Value of an Annuity of £1 after the Death of a Widower, and Payable until his Youngest Child reaches 21.

FATHER'S AGE.	SURVIVORSHIP ANNUITY OF £1.			FATHER'S AGE.
	3 per cent.	3½ per cent.	4 per cent.	
27	1·489	1·393	1·262	27
28	1·542	1·443	1·351	28
29	1·586	1·484	1·390	29
30	1·616	1·513	1·417	30
31	1·646	1·540	1·442	31
32	1·677	1·569	1·470	32
33	1·582	1·484	1·395	33
34	1·620	1·521	1·429	34
35	1·662	1·560	1·466	35
36	1·708	1·603	1·506	36
37	1·753	1·647	1·548	37
38	1·802	1·693	1·591	38
39	1·853	1·741	1·637	39
40	1·905	1·789	1·682	40
41	1·786	1·683	1·587	41
42	1·823	1·717	1·619	42
43	1·863	1·754	1·652	43
44	1·911	1·798	1·693	44
45	1·967	1·850	1·741	45
46	1·835	1·730	1·633	46
47	1·907	1·797	1·694	47
48	1·782	1·684	1·594	48
49	1·882	1·778	1·682	49
50	1·777	1·685	1·598	50
51	1·904	1·805	1·713	51
52	2·039	1·934	1·834	52
53	1·944	1·839	1·750	53
54	2·074	1·972	1·877	54
55	1·961	1·872	1·787	55
56	1·845	1·766	1·692	56
57	1·725	1·656	1·593	57
58	1·858	1·786	1·717	58
59	1·708	1·647	1·588	59
60	1·532	1·483	1·435	60
61	1·329	1·289	1·251	61
62	1·122	1·092	1·064	62
63	·918	·898	·877	63
64	·729	·714	·699	64
65	·551	·541	·532	65
66	·574	·563	·554	66
67	·407	·402	·396	67
68	·262	·259	·256	68
69	·140	·139	·138	69
70	·151	·151	·150	70

TABLE XIX.

Value of an Annuity of £1 after the Death of a Widow, and Payable
until the Youngest Child reaches 21.

MOTHER'S AGE.	SURVIVORSHIP ANNUITY OF £1.			MOTHER'S AGE.
	3 per cent.	3½ per cent.	4 per cent.	
20	1·187	1·110	1·039	20
21	1·217	1·138	1·065	21
22	1·253	1·172	1·096	22
23	1·292	1·209	1·131	23
24	1·337	1·250	1·170	24
25	1·385	1·295	1·212	25
26	1·319	1·238	1·163	26
27	1·370	1·285	1·208	27
28	1·421	1·334	1·253	28
29	1·462	1·373	1·289	29
30	1·491	1·400	1·315	30
31	1·519	1·426	1·339	31
32	1·548	1·453	1·365	32
33	1·582	1·485	1·397	33
34	1·491	1·403	1·322	34
35	1·530	1·440	1·357	35
36	1·572	1·480	1·394	36
37	1·614	1·521	1·433	37
38	1·659	1·562	1·473	38
39	1·562	1·475	1·395	39
40	1·600	1·512	1·429	40
41	1·482	1·405	1·333	41
42	1·505	1·425	1·355	42
43	1·375	1·307	1·243	43
44	1·396	1·326	1·261	44
45	1·424	1·352	1·285	45
46	1·294	1·233	1·174	46
47	1·335	1·270	1·210	47
48	1·212	1·159	1·106	48
49	1·103	1·056	1·012	49
50	1·000	·960	·922	50
51	1·077	1·034	·994	51
52	·977	·941	·906	52
53	·875	·846	·817	53
54	·771	·748	·725	54
55	·670	·652	·621	55
56	·571	·558	·544	56
57	·476	·466	·456	57
58	·381	·374	·368	58
59	·425	·417	·410	59
60	·313	·308	·304	60
61	·202	·199	·196	61
62	·106	·106	·105	62
63	·109	·109	·108	63
64	·112	·112	·111	64

Value of an Annuity of £1 after the Death of both Parents, and Payable until the Youngest Child attains the Age of 21 years.

WIFE'S AGE.	VALUE OF SURVIVORSHIP ANNUITY OF £1.			WIFE'S AGE.
	3 per cent.	3½ per cent.	4 per cent.	
20	·48549	·42401	·37158	20
21	·49552	·43414	·38182	21
22	·50558	·44436	·39202	22
23	·51523	·45424	·40197	23
24	·52504	·46446	·41231	24
25	·53506	·47476	·42280	25
26	·53734	·47798	·42675	26
27	·54126	·48277	·43217	27
28	·54423	·48672	·43686	28
29	·54565	·48927	·44028	29
30	·54509	·48995	·44201	30
31	·54346	·48970	·44289	31
32	·54055	·48829	·44311	32
33	·53632	·48569	·44155	33
34	·52994	·48111	·43847	34
35	·52456	·47741	·43624	35
36	·51637	·47112	·43160	36
37	·50604	·46283	·42523	37
38	·49367	·45265	·41695	38
39	·47890	·44027	·40670	39
40	·46522	·42881	·39559	40
41	·44853	·41454	·38344	41
42	·43320	·40145	·37227	42
43	·41485	·38551	·35844	43
44	·39817	·37105	·34594	44
45	·37779	·35308	·33010	45
46	·35601	·33376	·31300	46
47	·33517	·31533	·29663	47
48	·31052	·29315	·27665	48
49	·28743	·27225	·25781	49
50	·26596	·25280	·24017	50
51	·24456	·23327	·22233	51
52	·21527	·20610	·19707	52
53	·18393	·17673	·16954	53
54	·15061	·14518	·13968	54
55	·11823	·11451	·11025	55
56	·09677	·09388	·09106	56
57	·07759	·06548	·07346	57
58	·06036	·05888	·05749	58
59	·04458	·04359	·04105	59
60	·02364	·02319	·02273	60
61	·01651	·01624	·01596	61
62	·01163	·01149	·01131	62
63	·00882	·00873	·00862	63
64	·00527	·00523	·00518	64

TABLE XXI.

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Value of £1 payable at the Death of a Husband, if he die last and leave a Family.

DIFFERENCE OF AGE OF HUSBAND AND WIFE SEVEN YEARS.

WIFE'S AGE.	VALUE OF £1.			WIFE'S AGE.
	3 per cent.	3½ per cent.	4 per cent.	
20	·08154	·06579	·05345	20
21	·08381	·06789	·05534	21
22	·08627	·07013	·05738	22
23	·08891	·07254	·05954	23
24	·09167	·07509	·06188	24
25	·09455	·07774	·06430	25
26	·09678	·07984	·06625	26
27	·09906	·08199	·06826	27
28	·10132	·08415	·07028	28
29	·10346	·08621	·07222	29
30	·10542	·08811	·07403	30
31	·10741	·09005	·07588	31
32	·10944	·09205	·07778	32
33	·11163	·09418	·07984	33
34	·11402	·09652	·08207	34
35	·11655	·09899	·08443	35
36	·11910	·10147	·08683	36
37	·12165	·10399	·08925	37
38	·12421	·10654	·09173	38
39	·12677	·10910	·09423	39
40	·12921	·11155	·09665	40
41	·13136	·11377	·09887	41
42	·13322	·11575	·10089	42
43	·13491	·11758	·10279	43
44	·13649	·11934	·10465	44
45	·13814	·12116	·10658	45
46	·13990	·12311	·10864	46
47	·14180	·12521	·11084	47
48	·14389	·12748	·11322	48
49	·14635	·13011	·11596	49
50	·14914	·13306	·11901	50
51	·15233	·13639	·12242	51
52	·15581	·14002	·12611	52
53	·15977	·14408	·13023	53
54	·16443	·14881	·13496	54
55	·16947	·15391	·14007	55
56	·17464	·15918	·14555	56
57	·17967	·16451	·15073	57
58	·18494	·16971	·15603	58
59	·18922	·17419	·16065	59
60	·19333	·17759	·16426	60
61	·19379	·17945	·16643	61
62	·19461	·18070	·16804	62
63	·19510	·18166	·16937	63
64	·19549	·18250	·17062	64

Value of £1 payable at the Death of the Husband, if he die last
and leave a Family.

DIFFERENCE OF AGE OF HUSBAND AND WIFE SEVEN YEARS.

WIFE'S AGE.	VALUE OF £1.			WIFE'S AGE.
	3 per cent.	3½ per cent.	4 per cent.	
65	·19645	·18390	·17238	65
66	·19861	·18642	·17520	66
67	·20235	·19045	·17945	67
68	·20826	·19654	·18568	68
69	·21514	·20358	·19284	69
70	·22351	·21206	·20141	70
71	·23295	·22162	·21104	71
72	·24080	·22968	·21927	72
73	·24785	·23696	·22673	73
74	·25190	·24134	·23141	74
75	·25368	·24350	·23389	75
76	·25490	·24511	·23586	76
77	·25445	·24508	·23620	77
78	·25380	·24486	·23637	78
79	·25644	·24782	·23961	79
80	·25994	·25156	·24357	80

APPENDIX.

THE following specimens of the columns which may be employed in Valuation will be found to provide for all the information which it is usually considered necessary to collect.

MARRIED MEMBERS—STATISTICAL COLUMNS.

Age at Entry.	Ages Attained.		Ages at First Marriage.		Ages at Second Marriage.		Duration of First Married Life.	Age of First Wife at Death.	Age at First Wife's Death.	Duration of Widowhood.	Original Number of Children.				Number of Children surviving.		Ages of Children surviving.
	Husband.	Wife.	Husband.	Wife.	Husband.	Wife.					By First Marriage.	By Second Marriage.	Sons.	Daughters.	Sons.	Daughters.	

MARRIED MEMBERS—VALUATION COLUMNS.

VALUE OF £1 ANNUITY.					Value of £1 if Father die last and leave a Family.
Male Life.	Female Life.	Joint Life.	To Widows of Second Marriages.	To Children under Age at Death of last Parent.	

WIDOWERS—STATISTICAL COLUMNS.

Age at Entry.	Age at First Marriage.		Age at Second Marriage.		Duration of Married Life.		Wife's Age at Death.		Age at Wife's Death.		Years between First and Second Marriage.	Original Number of Children.				Number of Children surviving.	Ages of Children surviving.
	Husband.	Wife.	Husband.	Wife.	With First Wife.	With Second Wife.	First.	Second.	First.	Second.		By First Marriage.	Daughters.	Sons.	By Second Marriage.	Daughters.	

WIDOWERS—VALUATION COLUMNS.

VALUE OF £1 ANNUITY.				Value of £1 payable to Children at Father's Death.	Probability that the Widowers will not Marry again.
Widower's Life.	To Widows of Second Marriage.	To Children under Age after Father's Death.			